University Astronomy: Homework 14

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Question 20.1

At what distance (and at what redshift) does an object 1kpc across subtend an angle of 1 arcsecond?

$$\begin{aligned}
\theta'' &= \frac{206265d}{D} \\
1 &= \frac{206265 \times 1000}{D} \\
D &= 206265000pc \\
cz &= H_0 d \\
z &= \frac{H_0 d}{c} \\
&= 48128.5 \times 10^6
\end{aligned}$$

Question 20.2

The Ca_{II} H and K lines have rest wavelengths of $\lambda_0 = 3968.5$Å and 3933.6Å, respectively. In the spectrum of a galaxy in the cluster Abell 2065 (aka the Corona Borealis Cluster), the observed wavelengths of the two lines are $\lambda = 4255.0$Å and 4217.6Å.

(a) What is the redshift z of the galaxy?

$$\begin{aligned}
z &= \frac{\Delta\lambda}{\lambda} \\
&= \frac{4255\text{Å} - 3968.5\text{Å}}{4255\text{Å}} \\
&= 0.0673
\end{aligned}$$

(b) What is the distance to the galaxy?

$$\begin{aligned} cz &= H_0 d \\ d &= \frac{cz}{H_0} \\ &= 288.42 Mpc \end{aligned}$$

(c) What is the distance modulus of the galaxy?

$$\begin{aligned} M &= 5 \log \left(\frac{d}{10} \right) \\ &= 5 \log \left(\frac{388.42 \times 10^6 pc}{10 pc} \right) \\ &= 37.3 \end{aligned}$$

Question 20.3

Rewrite the relation for the distance modulus (equation 13.25) in terms of the redshift z rather than the distance d.

$$\begin{aligned} M &= 5 \log \left(\frac{d}{10} \right) \\ &= 5 \log \left(\frac{cz}{H_0} \frac{1}{10} \right) \\ &= 5 \log \left(\frac{cz}{10 H_0} \right) \end{aligned}$$

Question 20.6

Consider a black hole of mass $M = 10^8 M_\odot$. What is the maximum distance at which its radius of influence could be resolved using the *Hubble Space Telescope* at

a wavelength $\lambda \approx 1\mu m$?

$$\begin{aligned}
\log(\sigma_*) &\approx 2.2 \\
\sigma_* &\approx 10^{2.2}\frac{km}{s} = 10^{5.2}\frac{m}{s} \\
r_{bh} &= \frac{GM_{bh}}{\sigma_*^2} \\
&= 5.284 \times 10^{17} m \\
\theta &= \frac{r}{d} = 1.22\frac{\lambda}{D} \\
d &= \frac{rD}{1.22\lambda} \\
&= 1.04 \times 10^{24} m
\end{aligned}$$

Question 21.1

The quasar PDS 456 has a redshift $z = 0.184$ and an apparent magnitude $m_V = 14.0$.

(a) What is the distance to this quasar?

$$\begin{aligned}
cz &= H_0 d \\
d &= \frac{cz}{H_0} \\
&= 788.57 Mpc
\end{aligned}$$

(b) What is its absolute magnitude, M_V?

$$\begin{aligned}
m_V - M_V &= 5\log\left(\frac{d}{10}\right) \\
M_V &= m_V - 5\log\left(\frac{d}{10}\right) \\
&= -25.5
\end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech