University Astronomy: Homework 13

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Question 16.1

Compute $R = \frac{A_V}{E(B-B)}$, the ratio of total to selective absorption, for the case of Rayleigh scattering, $\tau \propto \lambda^{-4}$.

$$\begin{aligned} R &= \frac{A_V}{E(B-V)} \\ &= \frac{1}{(\frac{\tau_B}{\tau_B}) - 1} \\ &= \left(\frac{(\lambda_B)^{-4}}{(\lambda_V)^{-4}} - 1 \right)^{-1} \\ &= \left(\frac{4450^{-4}}{5500^{-4}} - 1 \right)^{-1} \\ &= 0.75 \end{aligned}$$

Question 16.2

The Sun emits 5×10^{23} photons per second with $h\nu > 13.6$ eV. If the density of hydrogen atoms in interplanetary space is $n = 10^9 m^{-3}$, what is the size of the Sun's Strögren sphere? Assume a recombination coefficient $\alpha = 2.6 \times 10^{-19} m^3 s^{-1}$.

$$\begin{aligned} R_S &= \left[\frac{3}{4\pi} \frac{Q_*}{\alpha(T_e) n_e^2} \right]^{\frac{1}{3}} \\ &= \left[\frac{3}{4\pi} \frac{5 \times 10^{23} s^{-1}}{2.6 \times 10^{-19} m^3 s^{-1} (10^9 m^{-3})^2} \right]^{\frac{1}{3}} \\ &= 77144093.9m = 7.71 \times 10^7 m \end{aligned}$$

Question 16.3

An A0 V star is observed to have $m_V = 14.0$ and $B - V = 1.5$. What is the distance to the star?

$$\begin{aligned} m - M &= 5\log(d) - 5 + A \\ &= 5\log(d) - 5 + R(B-V) \\ 14.0 - 0.65 &= 5\log(d) - 5 + (3.1)(1.5) \\ 13.7 &= 5\log(d) \\ d &= 10^{\frac{13.7}{5}} \\ &= 549.5pc \end{aligned}$$

Question 16.4

Dust grains made of graphite will sublime at a temperature $T \approx 1500K$. The albedo of graphite is $A \approx 0.04$.

1. How close to an O5 V star ($T_{eff} = 42000K, R = 12R_\odot$) can graphite grains survive?

$$\begin{aligned} L_{star} &= 4\pi R_{star}^2 \sigma_{SB} T_{star}^4 \\ E_{grain} &= \frac{L_{star}}{4\pi d^2}(4\pi R_{grain}^2)(1-A) \\ &= E_{radiated} = 4\pi R_{grain}^2 \sigma_{SB} T_{grain}^4 \\ \frac{4\pi R_{star}^2 \sigma_{SB} T_{star}^4}{4\pi d^2}(4\pi R_{grain}^2)(1-A) &= 4\pi R_{grain}^2 \sigma_{SB} T_{grain}^4 \\ \frac{R_{star}^2 T_{star}^4}{d^2}(1-A) &= 4T_{grain}^4 \\ d &= \left(\frac{4T_{grain}^4}{R_{star}^2 T_{star}^4 (1-A)}\right)^{-\frac{1}{2}} \\ &= 3.21 \times 10^9 m \end{aligned}$$

2. How close to an M2 III star ($T_{eff} = 3540K, R = 0.5R_\odot$) can graphite grains survive?

$$\begin{aligned} d &= \left(\frac{4T_{grain}^4}{R_{star}^2 T_{star}^4 (1-A)}\right)^{-\frac{1}{2}} \\ &= 9.49 \times 10^5 m \end{aligned}$$

Question 16.7

In general, an F0 main sequence star has absolute magnitude $M_V = 2.7$ and intrinsic color $(B-V)_0 = 0.30$. A specific F0 main sequence star is observed to have $m_V = 12.00$ and $m_B = 12.56$.

1. What is the color excess $E(B-V)$ for this star?

$$\begin{aligned}(B-V) &= (B-V)_0 + E(B-V)\\ E(B-V) &= (B-V) - (B-V)_0\\ &= (m_B - m_V) - (B-V)_0\\ &= (12.56 - 12.00) - 0.3\\ &= 0.26\end{aligned}$$

2. What is the extinction A_V for this star? (Assume $R = 3.1$).

$$\begin{aligned}R &= \frac{A_V}{E(B-V)}\\ A_V &= RE(B-V)\\ &= (3.1)(0.26)\\ &= 0.806\end{aligned}$$

3. What is the distance to this star?

$$\begin{aligned}m_V - M_V &= 5\log(d) - 5 + A_V\\ 5\log(d) &= m_V - M_V + 5 - A_V\\ d &= 10^{\frac{m_V - M_V + 5 - A_V}{5}}\\ &= 499.80pc\end{aligned}$$

4. What distance would you have computed if you had ignored extinction?

$$\begin{aligned}m_V - M_V &= 5\log(d) - 5\\ d &= 10^{\frac{m_V - M_V + 5}{5}}\\ &= 724.44pc\end{aligned}$$

Question 19.8

Assume that a galaxy is spherical. What radial dependence of the mass density $\rho(R)$ gives a flat rotation curve (that is, $\Theta(R) = \text{constant}$)? In this case, how does the enclosed mass $M(R)$ vary with radius R?

$$\begin{aligned}
\Theta(R) &= \sqrt{\frac{GM(R)}{R}} \\
M(R) &= \frac{\Theta(R)^2 R}{G} \\
&= \left(\frac{\Theta(R)^2}{G}\right) R \\
&\propto R \\
\rho(R) &= \frac{M(R)}{V} \\
&= \left(\frac{\Theta(R)^2}{G}\right) R \times \frac{3}{4\pi R^3} \\
&\propto \frac{1}{R^2}
\end{aligned}$$

Slide Question 1

What is the K-band extinction for a red giant that has observed $H - K = 2.35$? Note that $(H - K)_0 = 0.30$.

$$\begin{aligned}
E(H-K) &= (H-K) - (H-K)_0 \\
&= 2.35 - 0.30 \\
&= 2.05 \\
A(K) &= \frac{E(H-K)}{\frac{A(H)}{A(K)} - 1} \\
&= \frac{2.05}{(\frac{1.6}{2.2})^{-1.6} - 1} \\
&= 3.085
\end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech