University Astronomy: Homework 11

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Question 17.4

In this problem, you will estimate the duration of the horizontal branch phase in a $1M_\odot$ star.

1. Compute the energy released in the net triple alpha reaction $3^4\text{He} \rightarrow^{12}$ C. The masses of ^{4}He and ^{12}C are 4.0026 amu and 12.0000 amu, respectively, where 1 amu (atomic mass unit) $= 1.6606 \times 10^{-27}$kg.

$$\begin{aligned} E_1 &= (\Delta m)c^2 \\ &= (3 \times m_{He} + m_C)c^2 \\ &= 1.166 \times 10^{-12} J \end{aligned}$$

2. Assume that at the beginning of the horizontal branch phase, 10% of the original mass of the star is in the form of ^{4}He in the stellar core. Estimate the total energy released by fusing this amount of helium into carbon via the triple alpha process.

$$\begin{aligned} E_2 &= \frac{M_\odot}{10} \times \frac{E_1}{3m_{He}} \\ &= 1.163 \times 10^{43} J \end{aligned}$$

3. Assume that during the horizontal branch phase, $L = 100L_\odot$. If all this luminosity is provided by fusion of helium to carbon in the core, how long will the horizontal branch phase last?

$$\begin{aligned} L &= \frac{E_2}{t} \\ t &= \frac{E_2}{L} \\ &= 3.029 \times 10^{14} s = 9.604 \times 10^6 yr \end{aligned}$$

Question 18.1

What would the rotation period of the Sun if it collapsed to a radius $R = 6000km$ without losing angular momentum?

$$\begin{aligned} P &= \frac{2\pi R}{0.1c} \\ &= 1.26s \end{aligned}$$

Question 18.2

What is the radius of a $1.5M_\odot$ neutron star, expressed as a fraction of its Schwarzschild radius?

$$\begin{aligned} R_{ns} &= 11km \left(\frac{M_{ns}}{1.4M_\odot} \right)^{-\frac{1}{3}} \\ &= 10749.91m \\ R_{Sch} &= 3km \left(\frac{M_{ns}}{M_\odot} \right) \\ &= 4500m \\ \frac{R_{ns}}{R_{Sch}} &= 2.366 \end{aligned}$$

Question 18.4

What is the mean density of a $1.5M_\odot$ neutron star? A carbon nucleus has a radius $r \approx 3 \times 10^{-15}m$; what is its density? What is the ratio of the two densities?

$$\begin{aligned} R_{ns} &= 10749.91m \\ \rho_{ns} &= \frac{M_{ns}}{V_{ns}} = \frac{M_{ns}}{\frac{4}{3}\pi(R_{ns})^3} \\ &= 5.734 \times 10^{17} \frac{kg}{m^3} \\ \rho_C &= \frac{M_C}{V_C} = \frac{M_C}{\frac{4}{3}\pi(r_C)^3} \\ &= 1.762 \times 10^{17} \frac{kg}{m^3} \\ \frac{\rho_{ns}}{\rho_C} &= 3.25 \end{aligned}$$

Slide Question

How many stars with masses between 1 and 2 solar masses form in a star formation event if there are 10 stars with masses between 100 and 101 solar masses, assuming a Salpeter initial mass function?

$$\begin{aligned} 10 &= A \int_{100}^{101} m^{-2.35} \, \mathrm{d}m \\ &= A(1.972 \times 10^{-5}) \\ A &= \frac{10}{1.972 \times 10^{-5}} \\ &= 5.071 \times 10^{5} \\ N &= A \int_{1}^{2} m^{-2.35} \, \mathrm{d}m \\ &= 2.283 \times 10^{5} \end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech