University Astronomy: Homework 10

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Question 17.1

A protostellar cloud starts as a sphere of radius $R = 4000AU$ and temperature $T = 15K$. If it emits blackbody radiation, what is its total luminosity? What is the wavelength λ_p at which it emits the most radiation?

$$\begin{aligned} L &= 4\pi R^2 \sigma T^4 \\ &= 4\pi(5.9839 \times 10^{14} m)^2 (5.67 \times 10^{-8} W m^{-2} K^{-4})(15K)^4 \\ &= 1.29 \times 10^{28} W \\ \lambda_p &= \frac{(2.897 \times 10^{-3} \mu m K)}{T} \\ &= 182.33 \mu m \end{aligned}$$

Question 17.3

Consider two clouds in the interstellar medium. A molecular (H_2) cloud has $T = 10K$ and $n = 10^{12} m^{-3}$; a neutral atomic (H) cloud has $T = 120K$ and $n = 10^7 m^{-3}$. ($\gamma = \frac{7}{5}$ for the molecular cloud and $\gamma = \frac{5}{3}$ for the atomic cloud).

(a) What is the Jeans mass for each of the two clouds?
Molecular hydrogen (H_2):

$$\begin{aligned} \rho_0 &= 2m_p(10^{12} m^{-3}) \\ M_j &= 0.2 M_\odot \left(\frac{T}{10K}\right)^{\frac{3}{2}} \left(\frac{\rho_0}{3 \times 10^{-15} kg\ m^{-3}}\right)^{-\frac{1}{2}} \\ &= 3.767 \times 10^{29} kg \end{aligned}$$

Atomic hydrogen (H):

$$\begin{aligned}\rho_0 &= m_p(10^7 m^{-3}) \\ M_j &= 0.2M_\odot \left(\frac{T}{10K}\right)^{\frac{3}{2}} \left(\frac{\rho_0}{3\times 10^{-15} kg\ m^{-3}}\right)^{-\frac{1}{2}} \\ &= 7.002\times 10^{33} kg\end{aligned}$$

(b) What is the minimum radius each cloud must have to collapse? Molecular hydrogen (H_2):

$$\begin{aligned}\rho_0 &= 2m_p(10^{12} m^{-3}) \\ r_j &= \left(\frac{3\pi\gamma kT}{32G\rho_0\mu m_p}\right)^{\frac{1}{2}} \\ &= \left(\frac{3\pi\gamma kT}{32Gn(m_p)^2\mu}\right)^{\frac{1}{2}} \\ &= 3.904\times 10^{14} m\end{aligned}$$

Atomic hydrogen (H):

$$\begin{aligned}\rho_0 &= m_p(10^7 m^{-3}) \\ r_j &= \left(\frac{3\pi\gamma kT}{32G\rho_0\mu m_p}\right)^{\frac{1}{2}} \\ &= \left(\frac{3\pi\gamma kT}{32Gn(m_p)^2\mu}\right)^{\frac{1}{2}} \\ &= 6.599\times 10^{17} m\end{aligned}$$

(c) What is the timescale for the gravitational collapse of each cloud? Molecular hydrogen (H_2):

$$\begin{aligned}\rho_0 &= 2m_p(10^{12} m^{-3}) \\ t_{ff} &= \left(\frac{3\pi}{32G\rho_0}\right)^{\frac{1}{2}} \\ &= 1.149\times 10^{12} s = 36419.2 yr\end{aligned}$$

Atomic hydrogen (H):

$$\begin{aligned}\rho_0 &= m_p(10^7 m^{-3}) \\ t_{ff} &= \left(\frac{3\pi}{32G\rho_0}\right)^{\frac{1}{2}} \\ &= 5.136 \times 10^{14} s = 1.629 \times 10^7 yr\end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech