University Astronomy: Homework 9

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Question 15.1

What is the rate (in kilograms per second) at which the Sun is currently converting hydrogen to helium?

$$\begin{aligned}
\Delta m &= 4m_H - m_{He} \\
&= 4.83 \times 10^{-29} kg \\
E &= mc^2 \\
&= 4.347 \times 10^{-12} J \\
n_{reactions/s} &= \frac{L_\odot}{E} \\
&= 8.83 \times 10^{37} \frac{reactions}{s} \\
\delta M &= m n_{reactions/s} \\
&= 4.27 \times 10^{9} \frac{kg}{s}
\end{aligned}$$

Question 15.2

How much energy, in MeV, is produced per proton in the PP chain?

$$\begin{aligned}
\Delta m &= 4m_p - (m_{He} + 2m_{e^-}) \\
&= 4.26 \times 10^{-29} kg \\
E &= mc^2 \\
&= 3.83 \times 10^{-12} J = 23.92 MeV \\
\frac{23.92 MeV}{4p} &= 6.98 \frac{MeV}{p}
\end{aligned}$$

Question 15.3

Approximately half the original hydrogen in the Sun's core has now been converted to helium. Compute the mean molecular mass μ

1. at the surface of the sun, given standard abundances ($X_\odot = 0.734, Y_\odot = 0.250, Z_\odot = 0.016$)

$$\begin{aligned}\mu &= (2X + \frac{3}{4}Y + \frac{1}{2}Z)^{-1} \\ &= 0.6011\end{aligned}$$

2. at the center of the Sun

$$\begin{aligned}\mu &= (2X + \frac{3}{4}Y + \frac{1}{2}Z)^{-1} \\ X &= \frac{X_\odot}{2} \quad Y = Y_\odot + \frac{X_\odot}{2} \quad Z = Z_\odot \\ \mu &= 0.825\end{aligned}$$

Question 15.4

If a star has $M = 100M_\odot$ and $L = 10^6 L_\odot$, how long can it shine at that luminosity if it started as pure hydrogen and is able to convert all its H to He? If a star has $M = 0.5M_\odot$ and $L = 0.1L_\odot$, how long can it shine under the same conditions?

$$\begin{aligned}n_H &= \frac{100M_\odot}{m_p} \\ &= 3.055 \times 10^{55} \\ E &= n_H \Delta E \\ &= 1.2526 \times 10^{47} J \\ t &= \frac{E}{L} \\ &= 3.2628 \times 10^{14} s \approx 10.3 MYr\end{aligned}$$

A star with $M = 100M_\odot$ and $L = 10^6 L_\odot$ can shine for approximately 10 million years.

$$\begin{aligned} E &= \frac{0.5M_\odot}{m_p}\Delta E \\ &= 6.2640 \times 10^{44} J \\ t &= \frac{E}{L} \\ &= 5.17 \times 10^{-11} \text{ years} \end{aligned}$$

Question 15.5

Consider the Sun to be a sphere of uniform density that derives its luminosity from steady contraction. What fractional decrease in the Sun's radius, $\frac{\delta R}{R}$, would be required over historical times (say, the last 6000 years) to account for the Sun's constant luminosity over that period of time?

$$
\begin{aligned}
U &= -\frac{3}{5}\frac{Gm^2}{r} \\
m &= \rho V = \rho\left(\frac{4\pi r^3}{3}\right) \\
U &= -\frac{3}{5}\frac{G\rho}{r}\frac{16\pi^2 r^6}{9} \\
&= -\frac{16}{15}G\rho\pi^2 r^5 \\
L &= \frac{\mathrm{d}U}{\mathrm{d}r} \\
&= -\frac{16}{15}G\rho\pi^2 5r^4\ \mathrm{d}r \\
&= -\frac{16}{3}G\rho\pi^2 r^4\ \mathrm{d}r \\
&= \frac{U}{t} \\
\frac{\left(-\frac{16}{15}G\rho\pi^2 r^5\right)}{t} &= -\frac{16}{3}G\rho\pi^2 r^4\ \mathrm{d}r \\
\frac{1}{t}\left(\frac{1}{5}r\right) &= \ \mathrm{d}r \\
\frac{\mathrm{d}r}{r} &= \frac{1}{5t} \quad t = 6000 \text{ years} \\
\frac{\mathrm{d}r}{r} &= \frac{1}{30000} \text{ per year}
\end{aligned}
$$

Question 15.6

Suppose that the Sun is 100% carbon (coal, for instance) and that burning this can extract 3 eV per carbon nucleus. How long, assuming an inexhaustible supply of oxygen from outside, could burning carbon maintain the Sun's current luminosity?

$$
\begin{aligned}
E &= \frac{M_\odot}{12m_p}(3eV) \\
&= 4.8948 \times 10^{37} J \\
t &= \frac{E}{L} \\
&= 1.2750 \times 10^{-11} s = 4043.1 \text{ years}
\end{aligned}
$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech