University Astronomy: Homework 8

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Question 13.1

What is the apparent magnitude of the Sun as seen from Mercury at perihelion? What is the apparent magnitude of the Sun as seen from Eris at perihelion?

$$\begin{aligned} d_{Mercury} &= d_{major}\sqrt{1-e^2} \\ &= 0.378AU = 1.832 \times 10^{-6} pc \\ m_{Sun,mercury} &= M_{Sun,mercury} + 5\log(d_{Mercury}) - 5 \\ &= -28.855 \\ d_{Eris} &= d_{major}\sqrt{1-e^2} \\ &= 67.90AU = 0.00029pc \\ m_{Sun,Eris} &= M_{Sun,Eris} + 5\log(d_{Eris}) - 5 \\ &= -17.85 \end{aligned}$$

Question 13.2

Considering absolute magnitude M, apparent magnitude m, and distance d or parallax π'', compute the unknown for each of these stars:

(a) $m = -1.6$ mag, $d = 4.3$ pc. What is M?

$$M = m - 5\log(d) + 5 = 0.232 \text{ mag}$$

(b) $M = 14.3$ mag, $m = 10.9$ mag. what is d?

$$d = 10^{\frac{m-M+5}{5}} = 2.089 \text{ pc}$$

(c) $m = 5.6$ mag, $d = 88$ pc. What is M?

$$M = m - 5\log(d) + 5 = 0.877 \text{ mag}$$

(d) $M = -0.9$ mag, $d = 220$ pc. What is m?

$$m = M + 5\log(d) - 5 = 5.81 \text{ mag}$$

(e) $m = 0.2$ mag, $M = -9.0$ mag. What is d?

$$d = 10^{\frac{m-M+5}{5}} = 691.8$$

(f) $m = 7.4$ mag, $\pi'' = 0.0043''$. What is M?

$$d = \frac{1}{\pi''} = 232.55 \text{ pc}$$
$$M = m - 5\log(d) + 5 = 0.567 \text{ mag}$$

Question 13.3

What are the *angular diameters* of the following, as seen from the Earth?

(a) The Sun, with radius $R = R_\odot = 7 \times 10^5 km$

$$d = 2R_\odot = 14 \times 10^5 km$$
$$D_\theta \approx \tan^{-1}(\frac{d}{D}) \approx 0.009^\circ = 1930''$$

(b) Betelgeuse, with $M_V = -5.5$ mag, $m_V = 0.8$ mag, $R = 650R_\odot$

$$d = 2R = 1300R_\odot$$
$$D = 10^{\frac{m-M+5}{5}} = 181.97 \text{ pc} = 5.62 \times 10^{15} km$$
$$D_\theta \approx \tan^{-1}(\frac{d}{D}) \approx 0.033''$$

(c) The galaxy M31, with $R \approx 30$ kpc at a distance $D \approx 0.7$ Mpc

$$D_\theta \approx \tan^{-1}(\frac{d}{D}) = 4.899^\circ = 17636.7''$$

(d) The Coma cluster of galaxies, with $R \approx 3$ Mpc at a distance $D \approx 100$ Mpc

$$D_\theta \approx \tan^{-1}(\frac{d}{D}) = 3.43^\circ = 12361.1''$$

Question 13.4

The Lyten 726-8 star system contains two stars, one with apparent magnitude $m = 12.5$ and the other with $m = 12.9$. What is the combined apparent magnitude of the two stars?

$$
\begin{aligned}
m_{Vega} &= 0 \\
m_{Vega} - m_1 &= 2.5 \log(\frac{L_1}{L_{Vega}}) \\
-\frac{12,5}{2.5} &= \log(\frac{L_1}{L_{Vega}}) \\
\frac{L_1}{L_{Vega}} &= 10^{-\frac{12.5}{2.5}} \\
\frac{L_2}{L_{Vega}} &= 10^{-\frac{12.9}{2.5}} \\
\frac{L_{total}}{L_{Vega}} &= \frac{L_1 + L_2}{L_{Vega}} \\
m_{Vega} - m_{total} &= 2.5 \log(\frac{L_{total}}{L_{Vega}}) \\
-m_{total} &= -11.93 \\
m_{total} &= 11.93
\end{aligned}
$$

Question 13.5

A cluster of stars contains 100 stars with absolute magnitude $M = 0.0$, 1000 stars with $M = 3.0$, and 10000 stars with $M = 6.0$. What is the absolute magnitude of

the cluster taken as a whole?

$$
\begin{aligned}
m_{Vega} &= 0 \\
m_{Vega} - m &= 2.5 \log(\frac{L}{L_{Vega}}) \\
\frac{L_1}{L_{Vega}} &= 10^{-\frac{0}{2.5}} \\
\frac{L_2}{L_{Vega}} &= 10^{-\frac{3}{2.5}} \\
\frac{L_3}{L_{Vega}} &= 10^{-\frac{6}{2.5}} \\
\frac{L_{total}}{L_{Vega}} &= \frac{100L_1 + 1000L_2 + 10000L_3}{L_{Vega}} \\
-m_{total} &= 2.5 \log(\frac{L_{total}}{L_{Vega}}) \\
&= -5.76
\end{aligned}
$$

Question 13.8

The star Procyon A has an effective temperature $T_A = 6530K$ and a radius $R_A = 2.06R_\odot$. Its companion Procyon B has a radius $R_B = 0.0096R_\odot$ and an absolute bolometric magnitude $M_{bol,B} = 12.9$.

1. What is the ratio of the two objects' luminosities?

$$
\begin{aligned}
L_A &= 4\pi R_A^2 \sigma_{SB} T_A^4 \\
&= 2.659 \times 10^{27} W \\
L_B &= 10^{0.4(4.74 - M_{bol,B})} L_\odot \\
&= 2.090 \times 10^{23} W \\
\frac{L_A}{L_B} &= 1.2722 \times 10^4
\end{aligned}
$$

2. What is the ratio of their surface temperatures?

$$\begin{aligned} T_B &= \sqrt[4]{\frac{L_B}{4\pi(R_B)^2\sigma_{SB}}} \\ &= 9006.5K \\ \frac{T_A}{T_B} &= 0.73 \end{aligned}$$

Question 14.7

Consider the two stars whose properties are described below:

Star	V	$B-V$	M_V	$T_{eff}(K)$	Spectral Class	BC
Betelgeuse	0.45	1.50	-0.60	3370	M2 Ib	-1.62
Gliese 887	7.35	1.48	9.76	3520	M2 V	-1.89

How much larger in radius is Betelgeuse than Gliese 887?

$$\begin{aligned} M_{bol} &= BC + M_V \\ L &= 10^{0.4(4.74-M_{bol})}L_\odot \\ L_{Betelgeuse} &= 2.33 \times 10^{29} \\ L_{Gliese\ 877} &= 2.15 \times 10^{25} \\ \frac{R_{Betelgeuse}}{R_{Gliese\ 877}} &= \sqrt{\frac{L_{Betelgeuse}}{L_{Gliese\ 877}}}\left(\frac{T_{Gliese\ 877}}{T_{Betelgeuse}}\right)^2 \\ &= 113.6 \end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech