University Astronomy: Homework 5

Alvin Lin

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Question 3.2

The asteroid Eros is seen in opposition from the Earth once every 847 days. What is the sidereal orbital period of Eros? What is the length a of the semimajor axis of Eros' orbit? The eccentricity of the orbit of Eros is $e = 0.223$. Does Eros ever come within 1 AU of the Sun?

$$\begin{aligned}
\frac{1}{P} &= \frac{1}{P_E} - \frac{1}{P_{Eros}} \\
&= \frac{1}{365} - \frac{1}{847} \\
P &= 625\ days = 1.71yr \\
P^2 &= Ka^3 \\
a &= \sqrt[3]{\frac{P^2}{K}} \\
&= \sqrt[3]{\frac{1.71^2}{1yr^2 AU^{-3}}} \\
&= 1.431\ AU
\end{aligned}$$

$$
\begin{aligned}
e &= (1 - \frac{b^2}{a^2})^{\frac{1}{2}} \\
e^2 &= 1 - \frac{b^2}{a^2} \\
\frac{b^2}{a^2} &= 1 - e^2 \\
b &= \sqrt{a^2(1 - e^2)} \\
&= \sqrt{(1.431^2)(1 - 0.223^2)} \\
&= 1.395\ AU
\end{aligned}
$$

Eros does not come within 1 AU of the Sun.

Question 8.1

What is the mean mass density $\bar{\rho}$ of Saturn's largest satellite, Titan? What does this suggest about the composition of Titan?

$$
\begin{aligned}
\rho &= \frac{3M}{3\pi R^3} \\
&= \frac{3(1346 \times 10^{20}\ kg)}{4\pi(2575000\ m)^3} \\
&\approx 1882.97 \frac{kg}{m^3}
\end{aligned}
$$

This is typical of a Jovian planet, suggesting Titan is mostly gas and ice.

Question 8.2

Radioactive decay of elements in the Earth's interior results in a mean heat flux through the Earth's surface of $5 \times 10^{-2} W m^{-2}$. What is this flux expressed as a fraction of the energy flux due to thermal re-radiation of absorbed solar energy? If radioactive decay were the *only* heat source for the Earth, what would the Earth's

surface temperature be?

$$\begin{aligned}
F &= \frac{L_{Sun}}{4\pi r^2}(1-A) \\
&= \frac{3.839 \times 10^{26}\ W}{4\pi(1.496 \times 10^{8}\ km)^2}(1-0.4) \\
&= 8.19 \times 10^{8} W m^{-2} \\
\frac{5 \times 10^{-2} W m^{-2}}{8.19 \times 10^{8} W m^{-2}} &= 6.10500611 \times 10^{-11} \\
L_p &= 4\pi R^2 \sigma_{SB} T_p^4 \\
T_p &= \sqrt[4]{\frac{L_p}{4\pi R^2 \sigma_{SB}}} \\
&= \sqrt[4]{\frac{5 \times 10^{-2} W m^{-2}}{4\pi(6378000m)^2(5.670 \times 10^{-8} W m^{-2} K^{-4})}} \\
&= 0.0064K
\end{aligned}$$

Question 8.4

Pure, solid water ice has an albedo of $A \approx 0.35$. What is the minimum distance from the Sun at which a rapidly rotating ice cube would remain frozen? Between which two planets does this distance lie?

$$\begin{aligned}
T_p &\approx 395K(1-A)^{\frac{1}{4}}(\frac{r}{1AU})^{-\frac{1}{2}} \\
273.15K &= 395K(0.65)^{\frac{1}{4}}(\frac{r}{1AU})^{-\frac{1}{2}} \\
\frac{273.15K}{395K(0.65)^{\frac{1}{4}}} &= \sqrt{\frac{1AU}{r}} \\
\frac{1}{0.77} &= \frac{1}{r}AU \\
r &= 0.77AU
\end{aligned}$$

This distance lies between Venus and the Earth.

Question 10.5

How often does an observer at the Sun's location see the rings of Saturn exactly edge-on?

An observer at the Sun's location observes the rings of Saturn exactly edge-on twice per revolution of Saturn around the Sun (every 15 years).

Slide Question 1

What is the approximate energy of the Gamow peak?

300 keV

Slide Question 2

What is the process that generates energy for a star that has a central temperature of 25 million Kelvin?

The CNO cycle

Slide Question 3

How much gravitational energy will be generated as kinetic energy when the Sun shrinks to a white dwarf (roughly the size of the Earth)?

$$\begin{aligned} U &= \frac{3}{5}\frac{GM^2}{R} \\ &= \frac{3}{5}\frac{(6.673 \times 10^{-11} m^3 kg^{-1} s^{-2})(1.989 \times 10^{30} kg)^2}{6378 km} \\ &= 2.48 \times 10^{46} J \end{aligned}$$

Approximately half of this is converted to kinetic energy, so $\sim 1.24 \times 10^{46} J$.

Slide Question 4

What is the temperature in a shock induced by a relativistic jet of electrons accelerated to the speed of light?

$$\begin{aligned} T &= \frac{3}{16}\frac{mc^2}{k} \\ &= \frac{3}{16}\frac{(9.109 \times 10^{-31} kg)(2.998 \times 10^{8} m s^{-1})^2}{1.381 \times 10^{-23} m^2 kg s^{-2} K^{-1}} \\ &= 1.11 \times 10^{9} K \end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech