University Astronomy: Homework 4

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Question 5.1

Verify that the Maxwell-Boltzmann distribution has its maximum at a speed

$$\nu_p = \left(\frac{2kT}{m}\right)^{\frac{1}{2}}$$

$$\begin{aligned}
F(v)\ \mathrm{d}v &= 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 \mathrm{e}^{-\frac{mv^2}{2kT}}\ \mathrm{d}v \\
\frac{\mathrm{d}}{\mathrm{d}v}F(v) &= 0 \\
\frac{\mathrm{d}}{\mathrm{d}v}4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 \mathrm{e}^{-\frac{mv^2}{2kT}} &= 0 \\
Let: \alpha &= -\frac{m}{2kT} \\
\frac{\mathrm{d}}{\mathrm{d}v}v^2\mathrm{e}^{\alpha v^2} &= 0 \\
v^2\mathrm{e}^{\alpha v^2}2\alpha v + 2v\mathrm{e}^{\alpha v^2} &= 0 \\
\alpha v^2 + 1 &= 0 \\
v &= \pm\sqrt{-\frac{1}{\alpha}} \\
&= \pm\sqrt{-(-\frac{2kT}{m})} \\
v_{max} &= \sqrt{\frac{2kT}{m}} = \left(\frac{2kT}{m}\right)^{\frac{1}{2}}
\end{aligned}$$

Question 5.5

(a) A neutral sodium atom has ionization potential of $\chi = 5.1eV$. What is the speed of a free electron that has just barely enough kinetic energy to collisionally ionize a sodium atom in its ground state? What is the speed of a free *proton* with just enough kinetic energy to collisionally ionize this atom?

$$\begin{aligned}
\frac{1}{2}m_e v_e^2 &\geq \chi \\
v_e &\geq \sqrt{\frac{2\chi}{m_e}} \\
&\geq 1342278.88\frac{m}{s} \\
\frac{1}{2}m_p v_p^2 &\geq \chi \\
v_p &\geq \sqrt{\frac{2\chi}{m_p}} \\
&\geq 31329.96\frac{m}{s}
\end{aligned}$$

(b) What is the temperature T of a gas in which the average particle kinetic energy is just barely sufficient to ionize a sodium atom in its ground state?

$$\begin{aligned}
\langle E \rangle &= \frac{3}{2}kT \\
T &= \frac{2\chi}{3k} \\
&= 39443.15K
\end{aligned}$$

(c) At the temperature T computed in part (b), what is the expected thermal Doppler broadening, $\frac{\Delta\lambda}{\lambda}$, of a sodium spectral line? (Hint: the only stable isotope of sodium has mass number A = 23).

$$\begin{aligned}
\frac{\Delta\lambda}{\lambda} &\approx (3 \times 10^{-7})\left(\frac{T}{1K}\right)^{\frac{1}{2}} \mu^{-\frac{1}{2}} \\
&\approx (3 \times 10^{-7})\sqrt{\frac{39433.15K}{1K}}(23)^{-\frac{1}{2}} \\
&\approx 1.242 \times 10^{-5}
\end{aligned}$$

Question 5.8

If an incandescent light bulb has a luminosity $L = 60W$ and a filament temperature of $T = 2900K$, what must be the surface area of its filament? If the filament consists of a cylindrical wire with diameter $d = 4.6 \times 10^{-5}m$ (as in a standard incandescent 60 watt, 120 volt bulb), what is the length of the wire?

$$
\begin{aligned}
L &= A\sigma_{SB}T^4 \\
A &= \frac{L}{\sigma_{SB}T^4} \\
&= \frac{60W}{5.67 \times 10^{-8}Wm^{-2}K^{-4}(2900K)^4} \\
&= 1.496 \times 10^{-5}m^2 \\
&= \pi(\frac{d}{2})^2 l \\
l &= \frac{A}{\pi(\frac{d}{2})^2} \\
&= \frac{1.496 \times 10^{-5}}{\pi(\frac{4.6 \times 10^{-5}}{2})^2} \\
&= 9002.65m
\end{aligned}
$$

Slide Question 1

What is the signal to noise ratio for a source that generates 100 electrons in a detector if the observation is limited by shot noise from the source?

$$\frac{S}{N} = \frac{100}{\sqrt{100}} = \frac{100}{10} = 10$$

Slide Question 2

What is the equivalent width of the following absorption line? [diagram not shown]

$$\begin{aligned} W_\lambda &= \int (1 - \frac{F_\lambda}{F_0})\ \mathrm{d}\lambda \\ &= \int_{1.5\mu m}^{1.6\mu m} (1 - \frac{0.5}{1})\ \mathrm{d}\lambda \\ &= \int_{1.5\mu m}^{1.6\mu m} 0.5\ \mathrm{d}\lambda \\ &= 0.25\mu m \end{aligned}$$

Slide Question 3

What is the wavelength of the Bracket-gamma (HI 7-4) line?
  1458.93 nanometers.

Slide Question 4

In which waveband does the Bracket-gamma line appear?
  Infrared.

Slide Question 5

At which wavelength, observed by the DIRBE instrument on COBE, does zodiacal emission appear most prominent?
  Around 10 μm ($\sim 9.66\mu m$), or infrared wavelengths.

Slide Question 6

At which wavelength is the maximum flux for a blackbody of temperature 1000K?

$$\begin{aligned} \lambda_{max} T &= 2898\mu m K \\ \lambda_{max} &= \frac{2898\mu m}{1000} \\ &= 2.898\mu m = 2898nm \end{aligned}$$

(Infrared)

Slide Question 7

What is the wavelength of a photon emitted by a hydrogen atom with the electron in the ground state when it flips from a spin that is aligned with the spin of the proton to a spin that is anti-aligned with the spin of the proton?

The photon emitted has a 21 centimeter wavelength.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech