

University Astronomy: Homework 3

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January 2019 - May 2019

Question 5.1

Verify that the Maxwell-Boltzmann distribution has its maximum at a speed

$$v_p = \left(\frac{2kT}{m} \right)^{\frac{1}{2}}$$

$$F(v) \, dv = 4\pi \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} \, dv$$

$$\frac{d}{dv} F(v) = 0$$

$$\frac{d}{dv} 4\pi \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} = 0$$

$$\text{Let : } \alpha = -\frac{m}{2kT}$$

$$\frac{d}{dv} v^2 e^{\alpha v^2} = 0$$

$$v^2 e^{\alpha v^2} 2\alpha v + 2v e^{\alpha v^2} = 0$$

$$\alpha v^2 + 1 = 0$$

$$v = \pm \sqrt{-\frac{1}{\alpha}}$$

$$= \pm \sqrt{-\left(-\frac{2kT}{m}\right)}$$

$$v_{max} = \sqrt{\frac{2kT}{m}} = \left(\frac{2kT}{m} \right)^{\frac{1}{2}}$$

Question 5.5

- (a) A neutral sodium atom has ionization potential of $\chi = 5.1eV$. What is the speed of a free electron that has just barely enough kinetic energy to collisionally ionize a sodium atom in its ground state? What is the speed of a free *proton* with just enough kinetic energy to collisionally ionize this atom?

$$\begin{aligned} \frac{1}{2}m_e v_e^2 &\geq \chi \\ v_e &\geq \sqrt{\frac{2\chi}{m_e}} \\ &\geq 1342278.88 \frac{m}{s} \\ \frac{1}{2}m_p v_p^2 &\geq \chi \\ v_p &\geq \sqrt{\frac{2\chi}{m_p}} \\ &\geq 31329.96 \frac{m}{s} \end{aligned}$$

- (b) What is the temperature T of a gas in which the average particle kinetic energy is just barely sufficient to ionize a sodium atom in its ground state?

$$\begin{aligned} \langle E \rangle &= \frac{3}{2}kT \\ T &= \frac{2\chi}{3k} \\ &= 39443.15K \end{aligned}$$

- (c) At the temperature T computed in part (b), what is the expected thermal Doppler broadening, $\frac{\Delta\lambda}{\lambda}$, of a sodium spectral line? (Hint: the only stable isotope of sodium has mass number $A = 23$).

$$\begin{aligned} \frac{\Delta\lambda}{\lambda} &\approx (3 \times 10^{-7}) \left(\frac{T}{1K} \right)^{\frac{1}{2}} \mu^{-\frac{1}{2}} \\ &\approx (3 \times 10^{-7}) \sqrt{\frac{39443.15K}{1K}} (23)^{-\frac{1}{2}} \\ &\approx 1.242 \times 10^{-5} \end{aligned}$$

Question 5.8

If an incandescent light bulb has a luminosity $L = 60W$ and a filament temperature of $T = 2900K$, what must be the surface area of its filament? If the filament consists of a cylindrical wire with diameter $d = 4.6 \times 10^{-5}m$ (as in a standard incandescent 60 watt, 120 volt bulb), what is the length of the wire?

$$\begin{aligned}L &= A\sigma_{SB}T^4 \\A &= \frac{L}{\sigma_{SB}T^4} \\&= \frac{60W}{5.67 \times 10^{-8}Wm^{-2}K^{-4}(2900K)^4} \\&= 1.496 \times 10^{-5}m^2 \\&= \pi\left(\frac{d}{2}\right)^2l \\l &= \frac{A}{\pi\left(\frac{d}{2}\right)^2} \\&= \frac{1.496 \times 10^{-5}}{\pi\left(\frac{4.6 \times 10^{-5}}{2}\right)^2} \\&= 9002.65m\end{aligned}$$

Slide Question 1

What is the signal to noise ratio for a source that generates 100 electrons in a detector if the observation is limited by shot noise from the source?

$$\frac{S}{N} = \frac{100}{\sqrt{100}} = \frac{100}{10} = 10$$

Slide Question 2

What is the equivalent width of the following absorption line? [diagram not shown]

$$\begin{aligned}W_\lambda &= \int \left(1 - \frac{F_\lambda}{F_0}\right) d\lambda \\&= \int_{1.5\mu m}^{1.6\mu m} \left(1 - \frac{0.5}{1}\right) d\lambda \\&= \int_{1.5\mu m}^{1.6\mu m} 0.5 d\lambda \\&= 0.25\mu m\end{aligned}$$

Slide Question 3

What is the wavelength of the Bracket-gamma (HI 7-4) line?
1458.93 nanometers.

Slide Question 4

In which waveband does the Bracket-gamma line appear?
Infrared.

Slide Question 5

At which wavelength, observed by the DIRBE instrument on COBE, does zodiacal emission appear most prominent?

Around $10 \mu m$ ($\sim 9.66\mu m$), or infrared wavelengths.

Slide Question 6

At which wavelength is the maximum flux for a blackbody of temperature 1000K?

$$\begin{aligned}\lambda_{max}T &= 2898\mu mK \\ \lambda_{max} &= \frac{2898\mu m}{1000} \\ &= 2.898\mu m = 2898nm\end{aligned}$$

(Infrared)

Slide Question 7

What is the wavelength of a photon emitted by a hydrogen atom with the electron in the ground state when it flips from a spin that is aligned with the spin of the proton to a spin that is anti-aligned with the spin of the proton?

The photon emitted has a 21 centimeter wavelength.

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech