

# University Astronomy: Homework 3

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## Question 6.1

Assume that your vision is diffraction limited at  $\lambda = 5000\text{\AA}$  and that the diameter of the pupil of your eye is  $D = 8\text{mm}$ . What angular resolution can you achieve with your unaided eye? How does this compare with the maximum angular size of Venus and Jupiter as seen from Earth?

$$\theta = 1.22 \frac{5000\text{\AA}}{8\text{mm}} \times \frac{1000\text{mm}}{1\text{m}} \times \frac{1\text{m}}{10^{10}\text{\AA}} \times \frac{360^\circ}{2\pi \text{ rad}} \times \frac{3600''}{1^\circ} = 15.73''$$

The maximum angular size of Jupiter is  $50.1''$  and the maximum angular size of Venus is  $1'6''$ , larger than our minimum achieved angular resolution. We would be able to perceive both Venus and Jupiter with just our eye.

## Question 6.2

- (a) The Hiltner Telescope at the MDM Observatory (on Kitt Peak, Arizona) has an aperture  $D = 2.4\text{m}$ . Its Cassegrain focus has an f-number  $f/7$ . What is the focal length  $F$  and plate scale  $s$ .

$$\begin{aligned} \frac{F}{2.4} &= 7 \\ F &= 16.8\text{m} \\ s &= \frac{206.265}{16.8\text{m}} = 12.28 \frac{\text{arcsec}}{\text{mm}} \end{aligned}$$

- (b) The Mayall Telescope at the Kitt Peak National Observatory (also on Kitt Peak) has an aperture  $D = 4.0\text{m}$ . Its prime focus has an f-number  $f/2.7$ , its Cassegrain

focus has  $f/8$ , and its Coudé focus has  $f/160$ . What is the focal length and plate scale for each of these three foci?

$$\begin{aligned}
 F_1 &= Df_1 = (4)(2.7) = 10.8m \\
 s_1 &= \frac{206.265}{F} = \frac{206.265}{10.8} = 19.1 \frac{\text{arcsec}}{\text{mm}} \\
 F_2 &= Df_2 = (4)(8) = 32m \\
 s_2 &= \frac{206.265}{F} = \frac{206.265}{32} = 6.4 \frac{\text{arcsec}}{\text{mm}} \\
 F_3 &= Df_3 = (4)(160) = 320m \\
 s_3 &= \frac{206.265}{F} = \frac{206.265}{320} = 0.64 \frac{\text{arcsec}}{\text{mm}}
 \end{aligned}$$

- (c) The Keck Telescope (on Mauna Kea, Hawaii) has an aperture  $D = 10.0m$ . Its Cassegrain focus has  $f/15$ . What is the focal length and plate scale?

$$\begin{aligned}
 F &= Df = (10)(15) = 150m \\
 s &= \frac{206.265}{F} = \frac{206.265}{150} = 1.38 \frac{\text{arcsec}}{\text{mm}}
 \end{aligned}$$

### Question 6.3

With the  $D = 2.4m$  telescope at the MDM Observatory, I can obtain a spectrum of a particular star with signal-to-noise ratio  $S/N = 100$  in  $t = 20$  minutes when the atmospheric seeing is average  $\theta = 1''$ . How long would it take me to obtain the same data with the Keck Telescope ( $D = 10.0m$ ) with excellent seeing ( $\theta = 0.4''$ )?

Since the aperture size is 4.17 times larger and the seeing is 2.5 times better, the time would decrease proportionally so. It would take us  $\frac{20}{4.17 \times 2.5} = 1.92$  minutes to obtain the same data.

### Question 6.4

A charge-coupled device (CCD) detector is mounted at the focus of an  $f/7$  reflecting telescope with  $D = 50cm$  mirror. The CCD chip contains  $1024 \times 1024$  pixels, with each square pixel being  $10\mu m$  on a side.

- (a) What is the area (in square arcseconds) of the sky that is imaged on a single

pixel?

$$s = \frac{206.265}{7 \times 0.5} = 58.93 \frac{\text{arcsec}}{\text{mm}}$$

$$w = \frac{\sqrt{a}}{58.93} = 0.01 \text{mm}$$

$$a \approx (0.01 \times 58.93)^2 = 0.347 \text{ square arcseconds}$$

- (b) What is the area (in square arcminutes) of the sky that is imaged on the entire chip? Would the image of the full Moon fit into the chip?

$$a \approx 1024^2 \times 0.347 \times \left(\frac{1'}{60''}\right)^2$$

$$= 101.15 \text{ square arcminutes}$$

The maximum size of the Moon is 34 arcminutes, so the entire image would fit onto the chip.

- (c) How many separate exposures would be required to cover the entire celestial sphere ( $4\pi$  steradians)?

$$\frac{4\pi}{101.15 \text{ square arcminutes}} \times \left(\frac{180^\circ}{\pi}\right)^2 \times \left(\frac{60'}{1^\circ}\right)^2 \approx 1468966.8$$

### Question 6.5

Suppose that you want to see stars that are as faint as possible in the background limited case. The Astronomy Fairy gives you a choice: *either* she can increase the quantum efficiency of your retina from  $q = 0.1$  to  $q = 1$ , *or* she can double the maximum pupil size of your eye while guaranteeing diffraction-limited angular resolution. Which of these choices would produce a lower limited flux  $F_\lambda$ ? Explain your choice.

$$t \propto \left(\frac{\theta}{F_\lambda D \phi_a}\right)^2 \frac{S_\lambda}{\Delta \lambda \phi_t q}$$

$$\sqrt{t} \propto \frac{\theta}{F_\lambda D \phi_a} \sqrt{\frac{S_\lambda}{\Delta \lambda \phi_t q}}$$

$$F_\lambda \propto \frac{\theta}{D \phi_a} \sqrt{\frac{S_\lambda}{\Delta \lambda \phi_t q} \frac{1}{t}}$$

Doubling the maximum pupil size  $D$  would result in a proportional halving in  $F_\lambda$  (a factor of 0.5). Increasing the quantum efficiency from 0.1 to 1 would decrease  $F_\lambda$  by the square root of the change ( $\frac{1}{\sqrt{10}} = 0.316$ ). Increasing the quantum efficiency of your eye would produce a lower limited flux.

### Question 6.6

The Atacama Large Millimeter/Submillimeter Array (ALMA) is designed to operate over the wavelength range  $\lambda = 0.3 \rightarrow 9.6mm$ . It will consist of 80 independent 12m telescopes with a maximum baseline of 18km.

(a) What is the highest angular resolution achievable with ALMA?

$$\begin{aligned}\theta &= 1.22 \frac{\lambda}{D} \\ &= 1.22 \frac{9.6mm}{18000m} \times \frac{1m}{1000mm} \times \frac{180^\circ}{rad} \times \frac{3600''}{1^\circ} \\ &= 0.421''\end{aligned}$$

(b) How large would a single-dish antenna have to be to have the same collecting area as ALMA?

$$\begin{aligned}A_{total} &= 80\pi\left(\frac{12}{2}\right)^2 = 9043.2m^2 \\ &= \pi\left(\frac{D}{2}\right)^2 \\ D &= 2\sqrt{\frac{A_{total}}{\pi}} = 107.3m\end{aligned}$$

### Question 6.7

Prove that equation (6.18) is correct for a Poisson probability distribution.

$$\begin{aligned}\langle x^2 \rangle &= \sum_{x=0}^{\infty} x^2 P(x, \mu) \\ &= \sum_{x=1}^{\infty} x^2 \frac{\mu^x}{x!} e^{-\mu} \\ &= \sum_{x=1}^{\infty} x \frac{\mu^x e^{-\mu}}{(x-1)!} \\ z &= z-1 \quad x = z+1 \\ &= \sum_{z=0}^{\infty} (z+1) \frac{\mu^{z+1} e^{-\mu}}{z!} \\ &= \sum_{z=0}^{\infty} z \mu \frac{\mu^z e^{-\mu}}{z!} + \sum_{z=0}^{\infty} \mu \frac{\mu^z e^{-\mu}}{z!} \\ &= \mu \sum_{z=0}^{\infty} z \frac{\mu^z e^{-\mu}}{z!} + \mu \quad (\text{definition of } \langle x \rangle) \\ \sum_{z=0}^{\infty} z \frac{\mu^z e^{-\mu}}{z!} &= \mu \quad (\text{by analogy}) \\ &= \mu(\mu) + \mu \\ &= \mu^2 + \mu\end{aligned}$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)