University Astronomy: Homework 2

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Question 2.1

Over the course of the year, which gets more hours of daylight, the Earth's north pole or south pole?

The north pole gets more daylight over the course of the year because the south pole is closest to the Sun during perihelion, meaning it sweeps through that portion of the Earth's orbit around the Sun faster and spends less time in daylight.

Question 2.2

On 2003 August 27, Mars was in opposition as seen from the Earth. On 2005 July 14 (687 days later), Mars was seen in western quadrature as seen from the Earth. What was the distance of Mars from the Sun on these dates, measured in astronomical units (AU)? Is this greater than or less than the semimajor axis length of the Martian orbit? You may assume the Earth's orbit is a perfect circle.

After the 687 days (1.882 years), Mars is in the same position it was in before. However, the Earth has completed 1.882 rotations, meaning it is $(2 - 1.882) \times 360°$ behind Mars (the Mars-Sun-Earth angle). Since Mars is at quadrature, the Mars-Earth-Sun angle is $90°$. This forms a triangle and the Mars-Sun distance d can be described with the following relation:

$$d = \frac{1\ AU}{\cos((2 - 1.882)360°)} \approx 1.36\ AU$$

According to Kepler's third law, we can calculate the length of the semimajor axis length of the Martian orbit using $P^2 = Ka^3$.

$$\begin{aligned}
P^2 &= Ka^3 \\
(1.882\ yr)^2 &= \frac{1\ yr^2}{AU^{-3}} a^3 \\
3.54\ yr^2 &= \frac{1\ yr^2}{AU^{-3}} a^3 \\
a^3 &= 3.54 \frac{yr^2 AU^3}{yr^2} \\
a &= \sqrt[3]{3.54\ AU^3} \\
&= 1.52\ AU
\end{aligned}$$

This is greater than the calculated Mars-Sun distance.

Question 2.3

In the 1670s, the astronomer Ole Rømer observed eclipses of the Galilean satellite Io as it plunged through Jupiter's shadow once per orbit. He noticed that the time between observed eclipses became shorter as Jupiter came closer to the Earth and longer as Jupiter moved away from the. Rømer calculated that the eclipses were observed 17 minutes earlier when Jupiter was in opposition compared to when it was close to conjunction. This was attributed by Rømer to the finite speed of light. From Rømer's data, compute the speed of light, first in AU min^{-1}, then in m s^{-1}.

Let b be the Jupiter-Earth distance at conjunction and a be the Jupiter-Earth distance at opposition and c be the speed of light.

$$\begin{aligned}
b &= a + 2\ AU \\
a + c(17\ min) &= b \\
c &= \frac{b - a}{17\ min} \\
&= \frac{2\ AU}{17\ min} \\
&= 0.118 \frac{AU}{min} \\
&= 0.118 \frac{AU}{min} \times \frac{1\ min}{60\ s} \times \frac{149597780.7\ km}{1\ AU} \times \frac{1000\ m}{1\ km} \\
&= 294209145.71 \frac{m}{s}
\end{aligned}$$

Question 2.4

In addition to aberration of starlight due to the Earth's orbital motion around the Sun, there should also be diurnal aberration due to the Earth's rotation. Where on the Earth is this effect the largest, and what is the amplitude?

This effect would be largest at the Earth's equator, and would result in an angular correction of

$$\theta \approx \frac{v}{c} \approx \frac{0.465\ km\ s^{-1}}{3 \times 10^5\ km\ s^{-1}} \approx 0.23''$$

Question 2.5

A light-year is defined as the distance traveled by light in a vacuum during one tropical year. How many light-years are in a parsec?

$$1\ parsec \times \frac{206,205\ AU}{1\ parsec} \times \frac{1\ light-year}{63241.1\ AU} = 3.26\ light-years$$

Question 2.6

The planets of the solar system all orbit the Sun in the same sense; counterclockwise as seen from above the Earth's north pole. Imagine a "wrong-way" planet orbiting the Sun in the opposite (clockwise) sense, on an orbit of semimajor axis length $a = 1.3\ AU$. What would the sidereal period of this planet be? What would its synodic period be as seen from the Earth? What would its synodic period be as seen from Mars?

$$\begin{aligned}
P^2 &= Ka^3 \\
&= 1\frac{yr^2}{AU^{-3}}(1.3\ AU)^3 \\
P &= \sqrt{1.3^3\ yr^2} \\
&= 1.3^{\frac{3}{2}}\ yr \\
&= 1.482\ yr
\end{aligned}$$

The sidereal period of this planet would be 1.482 years. This planet would be considered a superior planet to Earth, reversing the angular velocity equation.

$$\begin{aligned}
\frac{1}{P_p} &= \frac{1}{P_E} + \frac{1}{P_{syn}} \\
P_{syn} &= \left[\frac{1}{P_p} + \frac{1}{P_E}\right]^{-1} \\
&= \left[\frac{1}{1.482\ yr} + \frac{1}{1\ yr}\right]^{-1} \\
&= 0.597\ yr
\end{aligned}$$

Relative to Earth, this planet would have a synodic period of 0.597 years. Relative to Mars, this would be an inferior planet.

$$\begin{aligned}
\frac{1}{P_p} &= \frac{1}{P_{Mars}} + \frac{1}{P_{syn}} \\
P_{syn} &= \left[\frac{1}{P_{Mars}} + \frac{1}{P_p}\right]^{-1} \\
&= \left[\frac{1}{1.882\ yr} + \frac{1}{1.482\ yr}\right]^{-1} \\
&= 0.829\ yr
\end{aligned}$$

As observed from Mars, this planet would have a synodic period of 0.829 years.

Question 2.7

Consider a football thrown directly northward at a latitude 40°N. The distance of the quarterback from the receiver is 20 yards (18.5m), and the speed of the thrown ball is 25m s^{-1}. Does the Coriolis effect deflect the ball to the right or to the left? By what amount (in meters) is the ball deflected? Does the receiver need to worry about correcting for the deflection, or should he be more worried about being nailed by the free safety?

The ball will be deflected to the right. It will have a flight time

$$\Delta t = \frac{18.5\ m}{25\ ms^{-1}} = 0.74\ s$$

Given this information:

$$\begin{aligned}\Delta d &\approx \frac{1}{2} vw(\delta t)^2 \\ &\approx \frac{1}{2}\frac{25\ m}{s}(\frac{1}{14000\ s})(0.74\ s)^2 \\ &\approx 0.000489\ m \\ &\approx 4.89 \times 10^{-4}\ m\end{aligned}$$

The amount of deflection is negligible to the receiver.

Slide Question 1

What is the name of the part of the eye that limits the amount of light that enters it?

The pupil is the aperture in the eye that admits light into it, and its size is adjusted by the iris.

Slide Question 2

Consider a photon with a vacuum wavelength of 550nm. What is its energy in a typical glass?

$$E = \frac{hc_m}{\lambda} = h\frac{c}{n_\lambda}\frac{1}{\lambda} = (6.6 \times 10^{-34} Js)\frac{3 \times 10^8 m/s}{1.5185}\frac{1}{550nm} = 2.37 \times 10^{-37} J$$

Slide Question 3

In which wavelength region is the atmosphere most transparent?

The atmosphere is most transparent to radio waves and visible light.

Slide Question 4

Use the Wien displacement law to calculate the wavelength for which the cosmic microwave background is at a peak, assuming that it is represented by a backbody with $T = 2.7K$.

$$\lambda_{max} = \frac{0.0029Km}{T} = \frac{0.0029Km}{2.7K} = 1.07mm$$

1mm microwave radiation.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech