University Physics 2

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Magnetic Fields

The magnetic force on a particle is given as:

$$\overrightarrow{F_B} = q(\vec{V} \times \vec{B})$$

Note that the force requires the charge to be moving and is perpendicular to both the velocity and the field since it is a cross product. Using the properties of cross products, we can express this as:

$$|\overrightarrow{F_B}| = q|\vec{V}||\vec{B}|\sin\theta$$

Cross Products

Recall the following properties of cross products:

1. Magnitude is a function of the vector magnitudes and the sine of the angle between them.
$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$

2. Direction follows the right-hand rule. Put your fingers in the direction of A, curl your fingers towards B, and your thumb will point towards C. With respect to its application to magnetic fields, we put our fingers in the direction of $\vec{V}$, curl them towards $\vec{B}$, and our thumb will point towards $\overrightarrow{F_B}$.

3. Using unit vectors:
$$\begin{aligned} \hat{i} \times \hat{j} &= \hat{k} & \hat{i} \times \hat{k} &= -\hat{j} \\ \hat{j} \times \hat{k} &= \hat{i} & \hat{k} \times \hat{j} &= -\hat{i} \\ \hat{k} \times \hat{i} &= \hat{j} & \hat{j} \times \hat{i} &= -\hat{k} \end{aligned}$$

Lorentz Force Law

Recall the following from electric fields:

$$\vec{F} = q\vec{E}$$

Thus, the net force of an electric and magnetic field is:

$$\begin{aligned}\vec{F} &= \overrightarrow{F_E} + \overrightarrow{F_B}\\ &= q\vec{E} + q(\vec{V} \times \vec{B})\\ &= q(\vec{E} + \vec{V} \times \vec{B})\end{aligned}$$

If we're dealing with magnetic fields only:

$$\begin{aligned}\overrightarrow{F_B} &= q(\vec{V} \times \vec{B})\\ |\overrightarrow{F_B}| &= q|\vec{V}||\vec{B}|\sin\theta\\ If: \theta &= 90\\ |\overrightarrow{F_B}| &= q|\vec{V}||\vec{B}|\end{aligned}$$

By the properties of cross products, we know that $\overrightarrow{F_B}$ is perpendicular to both $\vec{V}$ and $\vec{B}$. Recall that work $W = \vec{F} \cdot \delta\vec{r}$ is a dot product. Since $\Delta\vec{r}$ will be in the same direction as $\vec{V}$, $\overrightarrow{F_B} \cdot \Delta\vec{r} = 0$. Therefore, magnetic fields do no work.

Lorentz Force Law for a Wire

Moving charge is current, recall that:

$$I = nq|\vec{V}|A$$

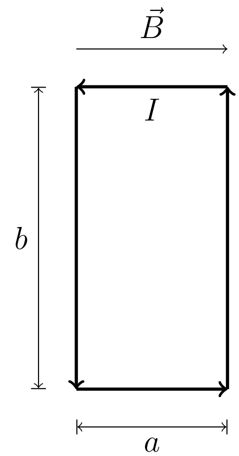
where n is charge density. The total number of charges in the wire is calculated as nAL where A is the cross-sectional area of the wire and L is the length of the wire. If the force on one charge passing through the wire is $|\vec{F}| = q|\vec{V}||\vec{B}|$, then the total force on all the charges passing through the wire is equal to:

$$\begin{aligned}|\vec{F_B}| &= (nAL)(q|\vec{V}||\vec{B}|)\\ &= (nq|\vec{V}|A)(L|\vec{B}|)\\ &= IL|\vec{B}|\\ \vec{F_B} &= I\vec{L} \times \vec{B}\end{aligned}$$

where $\vec{L}$ is a vector with the length of the wire, pointing in the direction of current. This is only true if the magnetic field is constant over the length of the wire. Otherwise, integration is necessary in order to calculate $\vec{F_B}$.

Example

What is the net force on the following loop of wire?



$$
\begin{aligned}
\vec{F} &= I\vec{L} \times \vec{B} \\
\overrightarrow{F_{top}} &= \overrightarrow{F_{bottom}} = \vec{0} \\
\overrightarrow{F_{total}} &= \overrightarrow{F_{top}} + \overrightarrow{F_{bottom}} + \overrightarrow{F_{left}} + \overrightarrow{F_{right}} \\
&= IbB\hat{k} + IbB(-\hat{k}) \\
&= 0
\end{aligned}
$$

Note that the net force from the magnetic field is zero, but there is a net torque exerted on the loop of wire.

$$
\begin{aligned}
\vec{\tau} &= \vec{r} \times \vec{F} \\
\overrightarrow{\tau_{top}} &= \overrightarrow{\tau_{bottom}} = \vec{0} \\
\overrightarrow{\tau_{total}} &= \overrightarrow{\tau_{top}} + \overrightarrow{\tau_{bottom}} + \overrightarrow{\tau_{left}} + \overrightarrow{\tau_{right}} \\
&= \frac{a}{2} IbB\hat{j} + \frac{a}{2} IbB\hat{j} \\
&= IabB\hat{j} \\
&= IAB\hat{j}
\end{aligned}
$$

where A is the area enclosed by the loop of wire. Note that $\mu = IA$ is known as the magnetic dipole moment.

Electric and Magnetic Dipoles

Recall from electric dipoles that the dipole moment points from the negative charge to the positive charge and has a magnitude of qd. We calculate torque for an electric dipole as $\overrightarrow{\tau_E} = \vec{p} \times \vec{E}$. For magnetic dipoles, an analogous concept applies. The torque on a magnetic dipole is $\overrightarrow{\tau_B} = \vec{\mu} \times \vec{B}$.

Magnetic Field Creation

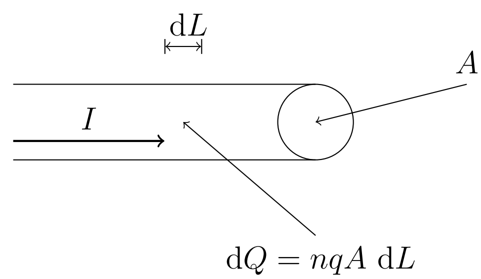
Magnetic fields all arise from current loops. Atoms are current loops as well. In physical materials, these current loops are evenly distributed and cancel out. For magnets, these current loops are aligned, creating a stronger magnetic field. An atom is the simplest current loop and the simply way to generate an magnetic field. Electric monopole can exists which propagate an electric field outwards in all direction. No analogous magnetic monopole exists since magnetic field lines always close on themselves. Magnetic fields are created by moving charge and defined as follows:

$$\vec{B} = \frac{\mu_\circ}{4\pi} \frac{q\vec{V} \times \vec{r}}{r^2}$$

where $\mu_\circ$ is the magnetic constant, q is the charge, $\vec{V}$ is the speed of the charge, r is the distance from the point of observation, and $\vec{r}$ is a unit vector pointing from the charge to the point of observation. Note that this formula for a magnetic field is analogous that of an electric field.

Biot-Savart Law

Current is just a bunch of moving charges in a wire, so magnetic fields are generated by current moving through a wire.



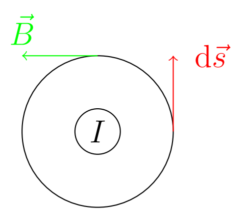
$$
\begin{aligned}
I &= nqA\vec{V} \\
\mathrm{d}\vec{B} &= \frac{\mu_\circ}{4\pi}\frac{\mathrm{d}Q\vec{V}\times\vec{r}}{r^2} \\
&= \frac{\mu_\circ}{4\pi}nqA\ \mathrm{d}\vec{L}\frac{\vec{V}\times\vec{r}}{r^2} \\
&= \frac{\mu_\circ}{4\pi}(nqA\vec{V})\frac{\mathrm{d}\vec{L}\times\vec{r}}{r^2} \\
&= \frac{\mu_\circ I}{4\pi}\frac{\mathrm{d}\vec{L}\times\vec{r}}{r^2} \\
\vec{B} &= \frac{\mu_\circ}{4\pi}\int I\frac{\mathrm{d}\vec{L}\times\vec{r}}{r^2}
\end{aligned}
$$

This relation is known as the Biot-Savart Law. All directions of magnetic fields come from using the right hand rule on the Biot-Savart Law where the fingers point in the direction of $\mathrm{d}\vec{s}$ and are curled in the direction of $\vec{r}$, thus pointing in the direction of $\mathrm{d}\vec{B}$.

Ampere's Law

$$\oint \vec{B}\cdot\ \mathrm{d}\vec{s} = \mu_\circ I_{enc}$$

Suppose we have a wire with current coming out of the page.



Ampere's Law is applied very similarly to Gauss's Law.

$$\oint \vec{B}\cdot\ \mathrm{d}\vec{s} = \int B\ \mathrm{d}s = B\int\ \mathrm{d}s = B2\pi r$$

This simplification can be made because of circular symmetry and because the magnetic field $\vec{B}$ is parallel to $\mathrm{d}\vec{s}$ around the wire.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech