

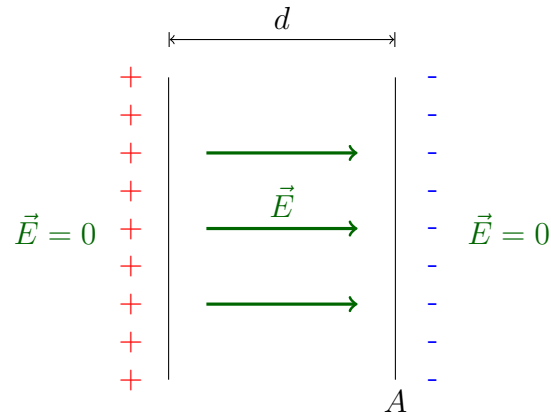
University Physics 2

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Capacitors

Capacitors are represented usually as two parallel plates.



We already know the following from electric fields and potential:

$$|\vec{E}_{capacitor}| = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \frac{Q}{A}$$

$$\Delta V = - \int_0^d \vec{E} \cdot d\vec{x} = - \int_0^d \frac{\sigma}{\epsilon_0} dx = - \frac{\sigma d}{\epsilon_0} = - \frac{Qd}{A\epsilon_0}$$

Capacitance of any object is defined as:

$$C \equiv \left| \frac{Q}{\Delta V} \right|$$

Capacitance is measured in units of Farads, with one Farad equivalent to one Coulomb over 1 volt. For parallel plates:

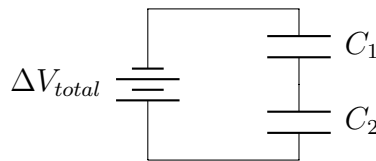
$$C_{parallel\ plates} = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{A\epsilon_0}} = \frac{A\epsilon_0}{d}$$

The assumes you have a vacuum between plates, generally this is not the case as there is some insulating material between the plates. Different materials determine the properties of the capacitor. Capacitance for this is:

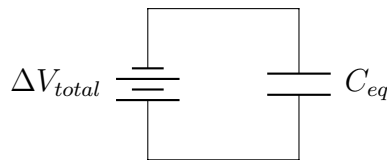
$$C = C_0\kappa = \kappa\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

where κ is the dielectric constant. $\kappa\epsilon_0$ is the permittivity of whatever material is used, and both constants are properties of the material. κ is always greater than 1, with $\kappa = 1$ being the dielectric constant for a vacuum.

Capacitors in Series



We can simplify this circuit to:



Both C_1 and C_2 have the same charge Q due to being in series with the battery.

$$Q_1 = Q_2 = Q_{total} \quad C_1 = \frac{Q_{total}}{\Delta V_1} \quad C_2 = \frac{Q_{total}}{\Delta V_2}$$

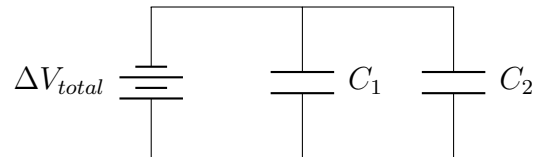
By conservation of energy, we know that:

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

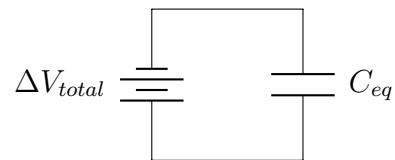
Thus, for capacitors in series, the equivalent capacitance can be calculated as:

$$\begin{aligned}
 C_{eq} &= \frac{Q_{total}}{\Delta V_{total}} \\
 Q_{total} &= C_{eq}(\Delta V_1 + \Delta V_2) \\
 Q_{total} &= C_{eq} \left(\frac{Q_{total}}{C_1} + \frac{Q_{total}}{C_2} \right) \\
 \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} \\
 C_{eq} &= \frac{C_1 C_2}{C_1 + C_2}
 \end{aligned}$$

Capacitors in Parallel



Again, we can simplify this circuit to:



Because the capacitors are in parallel:

$$C_1 = \frac{Q_1}{\Delta V_1} \quad C_2 = \frac{Q_2}{\Delta V_2}$$

By conservation of charge and conservation of energy:

$$Q_{total} = Q_1 + Q_2 \quad \Delta V_1 = \Delta V_{total} \quad \Delta V_2 = \Delta V_{total}$$

Thus, for capacitors in parallel, the equivalent resistance can be calculated as:

$$\begin{aligned}
 C_{eq} &= \frac{Q_{total}}{\Delta V} = \frac{Q_1 + Q_2}{\Delta V} \\
 &= \frac{Q_1}{\Delta V} + \frac{Q_2}{\Delta V} \\
 &= C_1 + C_2
 \end{aligned}$$

Energy in a Capacitor

To find the energy in a capacitor, find the work needed to charge it. Recall:

$$C = \frac{Q}{V} \quad V = \frac{Q}{C}$$

Suppose it is partially charged:

$$V = \frac{q}{C} \quad dV = \frac{dq}{C}$$

where V is the intermediate voltage and q is the intermediate charge.

$$\begin{aligned} W &= \int \vec{F} \cdot d\vec{r} & dW &= \vec{F} \cdot d\vec{r} \\ V &= - \int \vec{E} \cdot d\vec{r} & dV &= |\vec{E}| dr \\ dW &= F dr = (q|\vec{E}|) dr = q dV \\ W &= \int q dV = \int_0^Q q \left(\frac{dq}{C} \right) = \frac{Q^2}{2C} \end{aligned}$$

As a relation to the equations for voltage:

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

If we want energy density as energy over volume:

$$u = \frac{U}{\text{volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}(\frac{\epsilon_0 A}{d})(Ed)^2}{Ad} = \frac{1}{2}\epsilon_0 E^2$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech