University Physics 2

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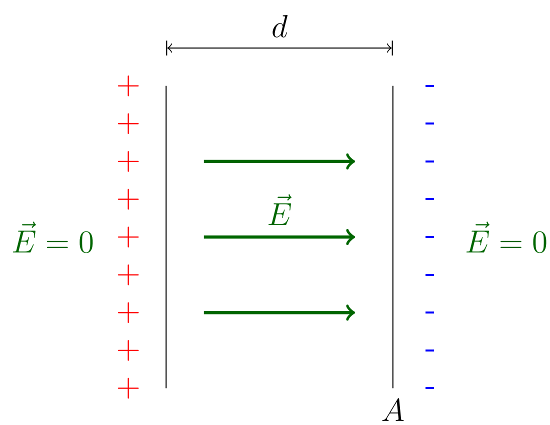
January 2018 - May 2018

Capacitors

Capacitors are represented usually as two parallel plates.



We already know the following from electric fields and potential:

$$|\overrightarrow{E_{capacitor}}| = \frac{\sigma}{\epsilon_\circ}$$

$$\sigma = \frac{Q}{A}$$

$$\Delta V = -\int_0^d \vec{E} \cdot \, \mathrm{d}\vec{x} = -\int_0^d \frac{\sigma}{\epsilon_\circ} \, \mathrm{d}x = -\frac{\sigma d}{\epsilon_\circ} = -\frac{Qd}{A\epsilon_\circ}$$

Capacitance of any object is defined as:

$$C \equiv \left|\frac{Q}{\Delta V}\right|$$

Capacitance is measured in units of Farads, with one Farad equivalent to one Coulomb over 1 volt. For parallel plates:

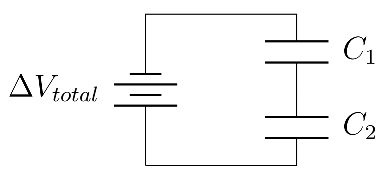
$$C_{parallel\ plates} = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{A\epsilon_\circ}} = \frac{A\epsilon_\circ}{d}$$

The assumes you have a vacuum between plates, generally this is not the case as there is some insulating material between the plates. Different materials determine the properties of the capacitor. Capacitance for this is:

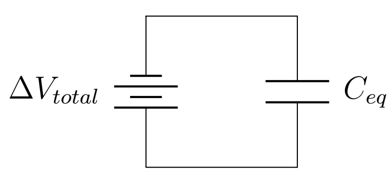
$$C = C_\circ\kappa = \kappa\epsilon_\circ\frac{A}{d} = \epsilon\frac{A}{d}$$

where κ is the dielectric constant. $\kappa\epsilon_\circ$ is the permitivity of whatever material is used, and both constants are properties of the material. κ is always greater than 1, with $\kappa = 1$ being the dielectric constant for a vacuum.

Capacitors in Series



We can simplify this circuit to:



Both C_1 and C_2 have the same charge Q due to being in series with the battery.

$$Q_1 = Q_2 = Q_{total} \quad C_1 = \frac{Q_{total}}{\Delta V_1} \quad C_2 = \frac{Q_{total}}{\Delta V_2}$$

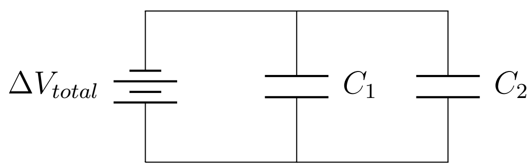
By conservation of energy, we know that:

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

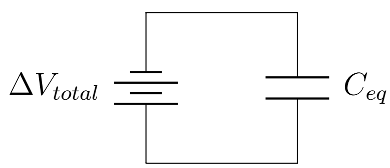
Thus, for capacitors in series, the equivalent capacitance can be calculated as:

$$\begin{aligned}
C_{eq} &= \frac{Q_{total}}{\Delta V_{total}} \\
Q_{total} &= C_{eq}(\Delta V_1 + \Delta V_2) \\
Q_{total} &= C_{eq}\left(\frac{Q_{total}}{C_1} + \frac{Q_{total}}{C_2}\right) \\
\frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} \\
C_{eq} &= \frac{C_1 C_2}{C_1 + C_2}
\end{aligned}$$

Capacitors in Parallel



Again, we can simplify this circuit to:



Because the capacitors are in parallel:

$$C_1 = \frac{Q_1}{\Delta V_1} \quad C_2 = \frac{Q_2}{\Delta V_2}$$

By conservation of charge and conservation of energy:

$$Q_{total} = Q_1 + Q_2 \quad \Delta V_1 = \Delta V_{total} \quad \Delta V_2 = \Delta V_{total}$$

Thus, for capacitors in parallel, the equivalent resistance can be calculated as:

$$\begin{aligned}
C_{eq} &= \frac{Q_{total}}{\Delta V} = \frac{Q_1 + Q_2}{\Delta V} \\
&= \frac{Q_1}{\Delta V} + \frac{Q_2}{\Delta V} \\
&= C_1 + C_2
\end{aligned}$$

Energy in a Capacitor

To find the energy in a capacitor, find the work needed to charge it. Recall:

$$C = \frac{Q}{V} \quad V = \frac{Q}{C}$$

Suppose it is partially charged:

$$V = \frac{q}{C} \qquad \mathrm{d}V = \frac{\mathrm{d}q}{C}$$

where V is the intermediate voltage and q is the intermediate charge.

$$W = \int \vec{F} \cdot \ \mathrm{d}\vec{r} \qquad \mathrm{d}W = \vec{F} \cdot \ \mathrm{d}\vec{r}$$
$$V = -\int \vec{E} \cdot \ \mathrm{d}\vec{r} \qquad \mathrm{d}V = |\vec{E}| \ \mathrm{d}r$$
$$\mathrm{d}W = F \ \mathrm{d}r = (q|\vec{E}|) \ \mathrm{d}r = q \ \mathrm{d}V$$
$$W = \int q \ \mathrm{d}V = \int_0^Q q(\frac{\mathrm{d}q}{c}) = \frac{Q^2}{2C}$$

As a relation to the equations for voltage:

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

If we want energy density as energy over volume:

$$u = \frac{U}{volume} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}(\frac{\epsilon_\circ A}{d})(Ed)^2}{Ad} = \frac{1}{2}\epsilon_\circ E^2$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech