University Physics 1A

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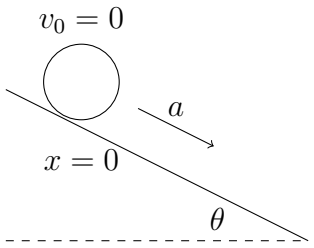
Torque

To calculate the acceleration of an object rolling down a ramp.



$$
\begin{aligned}
N - mg\cos\theta &= 0 \\
mg\sin\theta - f &= ma \\
fr &= \tau = I\alpha \\
a &= r\alpha \\
&= r\frac{fr}{I} \\
&= \frac{r^2}{I}(mg\sin\theta - ma) \\
a(1 + \frac{mr^2}{I}) &= \frac{r^2 mg\sin\theta}{I} \\
a &= \frac{r^2 mg\sin\theta}{I}\frac{1}{(1 + \frac{mr^2}{I})} \\
&= \frac{r^2 mg\sin\theta}{I + mr^2}
\end{aligned}
$$

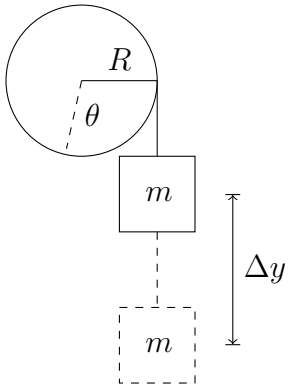
Using Conservation of Energy:

$$
\begin{aligned}
mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \\
mgh &= \left(\frac{1}{2}m + \frac{I}{2r^2}\right)v^2 \\
gh &= \frac{1}{2}(1 + \frac{I}{mr^2})v^2 \\
v^2 &= \frac{2gh}{1 + \frac{I}{mr^2}}
\end{aligned}
$$

Plugging into the kinematic equations:

$$
\begin{aligned}
v_f^2 &= v_i^2 + 2a(\Delta x) \\
\frac{2gh}{1 + \frac{I}{mr^2}} &= 2a\frac{h}{\sin\theta} \\
a &= \frac{g\sin\theta}{1 + \frac{I}{mr^2}} \\
&= \frac{mg\sin\theta}{m + \frac{I}{r^2}}
\end{aligned}
$$

Experiment



Find a formula for I in terms of α.

$$\begin{aligned}
\Delta y = s &= R\theta \\
-\frac{I\alpha}{R} - mg &= ma \\
&= mR\alpha \\
-\frac{I\alpha}{R} - mR\alpha &= mg \\
-\frac{I\alpha}{R} &= mg + mR\alpha \\
I &= -\frac{R}{\alpha}(mg + mR\alpha) \\
&= -\frac{mgR}{\alpha} - mR^2
\end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech