

# University Physics 1A

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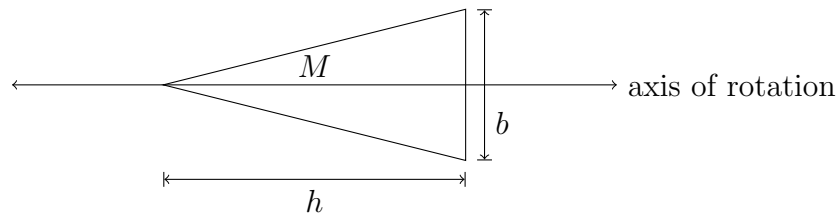
November 6th, 2017

## Moment of Inertia

Calculate  $I_{cm}$  for a flat, circular disk of mass  $M$  and radius  $R$  with a uniform mass distribution. Choose the axis to be perpendicular to the plane of the penny and passing through its center.

$$\begin{aligned}dA &= 2\pi r \, dr \\dm &= \sigma \, dA = \sigma 2\pi r \, dr \\&= \frac{M}{\pi R^2} 2\pi r \, dr \\I &= \int r^2 \, dm \\&= \int_0^R r^2 \frac{M}{\pi R^2} 2\pi r \, dr \\&= \frac{1}{2} MR^2\end{aligned}$$

Determine the moment of inertia of a thin triangular slab as show. Be careful with the limits of integration. It has mass  $M$  uniformly distributed, and dimensions as shown. There are still a couple of ways to make the slices, but the answer should be the same way either way.



$$\begin{aligned}
y &= \frac{b}{2h}x \\
dA &= (h - x) dy \\
&= \left(h - \frac{2h}{b}y\right) dy \\
dm &= \sigma dA \\
&= \frac{M}{\frac{1}{2}bh} \left(h - \frac{2h}{b}y\right) dy \\
I &= \int r^2 dm \\
&= 2 \int r^2 dm \\
&= 2 \int_0^{\frac{h}{2}} y^2 \frac{M}{\frac{1}{2}bh} \left(h - \frac{2h}{b}y\right) dy \\
&= \frac{1}{b} Mb^2
\end{aligned}$$

## Analogies between Linear and Rotational Motion

$$\begin{aligned}
KE &= \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 \\
KE &= \frac{1}{2}I\omega^2
\end{aligned}$$

Moment of inertia is analogous to mass.

$$F_{net} = ma$$

$$F_{net} = I\alpha$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)