University Physics 1A

Alvin Lin

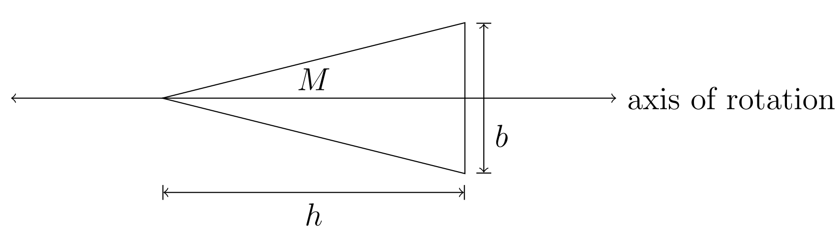
November 6th, 2017

Moment of Inertia

Calculate I_{cm} for a flat, circular disk of mass M and radius R with a uniform mass distribution. Choose the axis to be perpendicular to the plane of the penny and passing through its center.

$$
\begin{aligned}
\mathrm{d}A &= 2\pi r \ \mathrm{d}r \\
\mathrm{d}m &= \sigma \ \mathrm{d}A = \sigma 2\pi r \ \mathrm{d}r \\
&= \frac{M}{\pi r^2} 2\pi r \ \mathrm{d}r \\
I &= \int r^2 \ \mathrm{d}m \\
&= \int_0^R r^2 \frac{M}{\pi R^2} 2\pi r \ \mathrm{d}r \\
&= \frac{1}{2} M R^2
\end{aligned}
$$

Determine the moment of inertia of a thin triangular slab as show. Be careful with the limits of integration. It has mass M uniformly distributed, and dimensions as shown. There are still a couple of ways to make the slices, but the answer should be the same way either way.



$$
\begin{aligned}
y &= \frac{b}{2h}x \\
\mathrm{d}A &= (h - x)\ \mathrm{d}y \\
&= (h - \frac{2h}{b}y)\ \mathrm{d}y \\
\mathrm{d}m &= \sigma\ \mathrm{d}A \\
&= \frac{M}{\frac{1}{2}bh}(h - \frac{2h}{b}y)\ \mathrm{d}y \\
I &= \int r^2\ \mathrm{d}m \\
&= 2\int r^2\ \mathrm{d}m \\
&= 2\int_0^{\frac{h}{2}} y^2 \frac{M}{\frac{1}{2}bh}(h - \frac{2h}{b}y)\ \mathrm{d}y \\
&= \frac{1}{b}Mb^2
\end{aligned}
$$

Analogies between Linear and Rotational Motion

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2$$

$$KE = \frac{1}{2}I\omega^2$$

Moment of inertia is analogous to mass.

$$F_{net} = ma$$

$$F_{net} = I\alpha$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech