

# University Physics 1A

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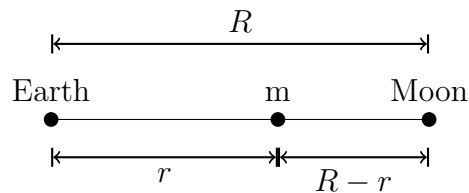
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## Gravitational Force

$$F = \frac{Gm_1m_2}{R^2}$$

$$U_g = -\frac{Gm_1m_2}{r}$$

At what point between the Earth and the Moon will a particle experience the same force of gravitational attraction towards both bodies?



$$\begin{aligned} F_{Earth} &= F_{Moon} \\ G \frac{m_E m}{r^2} &= G \frac{m_{Moon} m}{(R - r)^2} \\ \frac{(R - r)^2}{r^2} &= \frac{m_{Moon}}{m_{Earth}} \\ \frac{R - r}{r} &= \pm \sqrt{m_{Moon} m_{Earth}} \\ r &= 3.43 \times 10^5 km \\ r &= 4.27 \times 10^4 km \end{aligned}$$

## Escape Velocity

Escape speed,  $v_{esc}$  (escape velocity) is defined as the speed with which an object must be launched from the surface of a massive body (such as a planet) so that it just barely escapes the planet (neglecting any air resistance). Find the speed at which a mass  $m$  needs to be launched in order to escape the gravity of a planet of size  $R$ .



Using the equations for mechanical energy:

$$\frac{1}{2}m(v_0)^2 - \frac{Gm_p m}{R} = \frac{1}{2}m(0)^2 - 0$$

$$v_0 = \sqrt{\frac{2Gm_p}{R}}$$

### Example

You are an astronaut standing on the surface of the asteroid Ceres. Ceres has a mass of  $9.4 \times 10^{20}$  kg and a radius of 470 km. You use a catapult to launch a small rock upwards with an initial speed of 300.0 m/s. What is the maximum height that the rock reaches?

$$\begin{aligned} KE + PE &= PE \\ \frac{1}{2}m(v_0)^2 + \left(\frac{-Gm_C m}{R}\right) &= \frac{-Gm_C m}{R+h} \\ \frac{1}{2}(v_0)^2 - \frac{Gm_C}{R} &= \frac{-Gm_C}{R+h} \\ R+h &= \frac{-Gm_C}{\frac{1}{2}v_0^2 - \frac{Gm_C}{R}} \\ h &= \frac{-Gm_C}{\frac{1}{2}v_0^2 - \frac{Gm_C}{R}} - R \\ &= 239 \text{ km} \end{aligned}$$

What initial speed would the rock need to totally escape the asteroid?

$$\begin{aligned}v_0 &= \sqrt{\frac{2Gm_C}{R}} \\&= \sqrt{\frac{2(6.67 \times 10^{-11})(9.4 \times 10^{20})}{470000}} \\&= 516.52 \text{ m/s}\end{aligned}$$

### Practice Problem

Consider the following situation. A bullet of mass  $m_1$  is shot with a speed of  $v_1$  towards a block of mass  $M_2$  that is at rest and attached to a relaxed spring of constant  $k$ . The bullet enters the block and embeds inside of it. The block moves on the frictionless surface making the spring compress by a maximum amount  $X$ . What is  $X$ ?

$$m_1v_1 + M_2v_2 = (m_1 + M_2)v_f$$

$$m_1v_1 = (m_1 + M_2)v_f$$

$$v_f = \frac{m_1v_1}{m_1 + M_2}$$

$$KE = PE_s$$

$$\frac{1}{2}(m_1 + M_2)v_f^2 = \frac{1}{2}kX^2$$

$$X^2 = \frac{(m_1 + M_2)v_f^2}{k}$$

$$X = \sqrt{\frac{(m_1 + M_2)v_f^2}{k}}$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)