University Physics 1A

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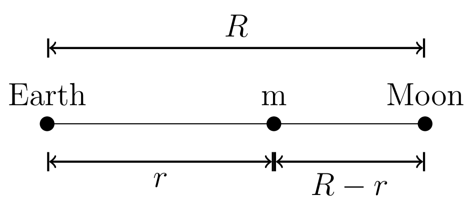
October 26th, 2017

Gravitational Force

$$F = \frac{Gm_1m_2}{R^2}$$
$$U_g = -\frac{Gm_1m_2}{r}$$

At what point between the Earth and the Moon will a particle experience the same force of gravitational attraction towards both bodies?



$$\begin{aligned}
F_{Earth} &= F_{Moon} \\
G\frac{m_E m}{r^2} &= G\frac{m_{Moon} m}{(R-r)^2} \\
\frac{(R-r)^2}{r^2} &= \frac{m_{Moon}}{m_{Earth}} \\
\frac{R-r}{r} &= \pm\sqrt{m_{Moon}}m_{Earth} \\
r &= 3.43 \times 10^5 km \\
r &= 4.27 \times 10^4 km
\end{aligned}$$

Escape Velocity

Escape speed, v_{esc} (escape velocity) is defined as the speed with which an object must be launched from the surface of a massive body (such as a planet) so that it just barely escapes the planet (neglecting any air resistance). Find the speed at which a mass m needs to be launched in order to escape the gravity of a planet of size R.

Using the equations for mechanical energy:

$$\frac{1}{2}m(v_0)^2 - \frac{Gm_p m}{R} = \frac{1}{2}m(0)^2 - 0$$

$$v_0 = \sqrt{\frac{2Gm_p}{R}}$$

Example

You are an astronaut standing on the surface of the asteroid Ceres. Ceres has a mass of 9.4×10^{20} kg and a radius of 470 km. You use a catapult to launch a small rock upwards with an initial speed of 300.0 m/s. What is the maximum height that the rock reaches?

$$\begin{aligned} KE + PE &= PE \\ \frac{1}{2}m(v_0)^2 + (\frac{-Gm_C m}{R}) &= \frac{-Gm_C m}{R+h} \\ \frac{1}{2}(v_0)^2 - \frac{Gm_C}{R} &= \frac{-Gm_C}{R+h} \\ R+h &= \frac{-Gm_C}{\frac{1}{2}v_0^2 - \frac{Gm_c}{R}} \\ h &= \frac{-Gm_C}{\frac{1}{2}v_0^2 - G\frac{m_c}{R}} - R \\ &= 239\ km \end{aligned}$$

What initial speed would the rock need to totally escape the asteroid?

$$\begin{aligned} v_0 &= \sqrt{\frac{2Gm_C}{R}} \\ &= \sqrt{\frac{2(6.67 \times 10^{-11})(9.4 \times 10^{20})}{470000}} \\ &= 516.52\ m/s \end{aligned}$$

Practice Problem

Consider the following situation. A bullet of mass m_1 is shot with a speed of v_1 towards a block of mass M_2 that is at rest and attached to a relaxed spring of constant k. The bullet enters the block and embeds inside of it. The block moves on the frictionless surface making the spring compress by a maximum amount X. What is X?

$$\begin{aligned} m_1v_1 + M_2v_2 &= (m_1 + M_2)v_f \\ m_1v_1 &= (m_1 + M_2)v_f \\ v_f &= \frac{m_1v_1}{m_1 + M_2} \\ KE &= PE_s \\ \frac{1}{2}(m_1 + M_2)v_f^2 &= \frac{1}{2}kX^2 \\ X^2 &= \frac{(m_1 + M_2)v_f^2}{k} \\ X &= \sqrt{\frac{(m_1 + M_2)v_f^2}{k}} \end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech