

Boundary Value Problems: Homework 7

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Problem 1

Determine whether the following PDE is separable or not.

$$\begin{aligned}u_{xx} + u_{xt} + u_t &= 0 \\ \frac{\partial}{\partial x^2}(X(x)T(t)) + \frac{\partial}{\partial x \partial t}(X(x)T(t)) + \frac{\partial}{\partial t}(X(x)T(t)) &= 0 \\ X''(x)T(t) + X'(x)T'(t) + X(x)T'(t) &= 0 \\ T'(t)(X'(x) + X(x)) &= -X''(x)T(t) \\ \frac{T'(t)}{T(t)} &= -\frac{X''(x)}{X'(x) + X(x)}\end{aligned}$$

Problem 2

Determine whether the following PDE is separable or not.

$$\begin{aligned}u_{xx} + (x + y)u_{yy} &= 0 \\ \frac{\partial}{\partial x^2}(X(x)Y(y)) + (x + y)\frac{\partial}{\partial y^2}(X(x)Y(y)) &= 0 \\ X''(x)Y(y) + (x + y)X(x)Y''(y) &= 0\end{aligned}$$

Not separable.

Problem 3

Determine whether the following PDE is separable or not.

$$\begin{aligned}u_{xy} - u &= 0 \\ \frac{\partial}{\partial x \partial y}(X(x)Y(y)) - X(x)Y(y) &= 0 \\ X'(x)Y'(y) &= X(x)Y(y) \\ \frac{X'(x)}{X(x)} &= \frac{Y(y)}{Y'(y)}\end{aligned}$$

Problem 4

Determine whether the following PDE is separable or not.

$$\begin{aligned}u_{tt} - u_{xx} &= 0 \\ \frac{\partial}{\partial t^2}(T(t)X(x)) - \frac{\partial}{\partial x^2}(T(t)X(x)) &= 0 \\ T''(t)X(x) &= T(t)X''(x) \\ \frac{T''(t)}{T(t)} &= \frac{X''(x)}{X(x)}\end{aligned}$$

Problem 5

Determine whether the following PDE is separable or not.

$$\begin{aligned}t^2 u_{tt} - x^2 u_{xx} &= 0 \\ t^2 \frac{\partial}{\partial t^2}(X(x)T(t)) - x^2 \frac{\partial}{\partial x^2}(X(x)T(t)) &= 0 \\ t^2 X(x)T''(t) &= x^2 X''(x)T(t) \\ \frac{t^2 T''(t)}{T(t)} &= \frac{x^2 X''(x)}{X(x)}\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech