Boundary Value Problems: Homework 4

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Problem 1

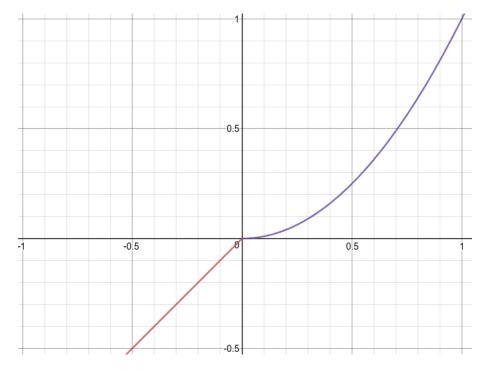
Exercises 3.1: problems 1, 2

Exercise 1

Graph the function

$$f(x) = \begin{cases} x & , x < 0\\ 1 & , x = 0\\ x^2 & , x > 0 \end{cases}$$

Determine whether the function is PWC, continuous, PWS, and smooth.



This function is piecewise continuous and piecewise smooth.

Exercise 2

If f(x) = |x|, is f(x) continuous at x = 0?

$$\lim_{x \to 0_+} f(x) = \lim_{x \to 0_-} f(x) = \lim_{x \to 0} f(x) = 0$$

f(x) is continuous at 0.

$$f'(0_+) = 1$$

 $f'(0_-) = -1$

f(x) is not differentiable at 0 because the left and right hand derivatives differ on both sides.

Problem 2

Determine whether each of the following functions is piecewise smooth.

(a)

$$f(x) = \begin{cases} -x & , -\pi \le x < 0\\ 1 & , 0 \le x \le \pi \end{cases}$$
$$f'(x) = \begin{cases} -1 & , \pi \le x < 0\\ 0 & , 0 \le x \le \pi \end{cases}$$

This function is piecewise smooth.

(b)

$$f(x) = \begin{cases} 2 & ,-5 \le x < 0\\ \frac{1}{x-1} & ,0 \le x \le 5 \end{cases}$$
$$f'(x) = \begin{cases} 0 & ,-5 \le x < 0\\ \ln(x-1) & ,0 \le x \le 5 \end{cases}$$

This function is not piecewise smooth.

Problem 3

Suppose $C_n = (1 - \cos(n\pi))$, where $n \in \mathbb{N}$. Find expressions for C_{2n-1} and C_{2n} and simply the expressions and much as possible. The simplified expressions should just involve numbers, rather than cosines.

$$C_{2n-1} = 1 - \cos((2n - 1)\pi)$$

= 1 - (-1)
= 2 (2n-1 is an odd number)
$$C_{2n} = 1 - \cos(2n\pi)$$

= 1 - (1)
= 0 (2n is an even number)

Problem 4

Find the coefficients $a_0, a_n, b_n (n \in \mathbb{N})$, of the Fourier series for

$$f(x) = \begin{cases} 1 & , -2 \le x < 0 \\ 0 & , 0 < x < 2 \end{cases}$$

$$a_{0} = \frac{1}{L} \int_{-L}^{L} f(x) dx$$

$$= \frac{1}{2} (\int_{-2}^{0} 1 dx + \int_{0}^{2} 0 dx)$$

$$= \frac{1}{2} (2 + 0)$$

$$= 1$$

$$a_{n} = \frac{1}{L} \int_{-L}^{L} f(x) \cos(\frac{n\pi x}{L}) dx$$

$$= \frac{1}{2} \left(\int_{-2}^{0} 1 \cos(\frac{n\pi x}{2}) dx + \int_{0}^{2} 0 dx \right)$$

$$= \frac{1}{2} \frac{2}{n\pi} \left[\sin(\frac{n\pi x}{2}) \right]_{-2}^{0}$$

$$= \frac{1}{n\pi} (\sin(-n\pi) - \sin(0))$$

$$= 0$$

$$b_{n} = \frac{1}{2} \int_{-L}^{L} f(x) \sin(\frac{n\pi x}{L}) dx$$

$$= \frac{1}{2} \left(\int_{-2}^{0} 1 \sin(\frac{n\pi x}{2}) dx + \int_{0}^{2} 0 dx \right)$$

$$= \frac{1}{2} \frac{2}{n\pi} \left[-\cos(\frac{n\pi x}{2}) \right]_{0}^{2}$$

$$= \frac{1}{n\pi} (-\cos(n\pi) + \cos(0))$$

$$= \frac{1 - \cos(n\pi)}{n\pi}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech