Boundary Value Problems: Homework 1

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Problem 1

Use the definition of a linear operator (given in Equation 1.1 in your textbook) to show that the operator L as defined $Ly = 3Dy - 2y$ is a linear operator. Here D is the first-derivative operator.

$$\begin{aligned}
L(c_1y_1 + c_2y_2) &= c_1Ly_1 + c_2Ly_2 \\
3D(c_1y_1 + c_2y_2) - 2(c_1y_1 + c_2y_2) &= c_1(3Dy_1 - 2y_1) + c_2(3Dy_2 - 2y_2) \\
3c_1Dy_1 + 3c_2Dy_2 - 2c_1y_1 - 2c_2y_2 &= 3c_1Dy_1 - 2c_1y_1 + 3c_2Dy_2 - 2c_2y_2
\end{aligned}$$

Problem 2

Use the definition of a lienar operator (given in Equation 1.1 of your textbook) to show that the squaring operator, Q, defined as $Qy = y^2$ is not linear.

$$\begin{aligned}
L(y_1 + y_2) &= Ly_1 + Ly_2 \\
Q(y_1 + y_2) &= Qy_1 + Qy_2 \\
(y_1)^2 + 2y_1y_2 + (y_2)^2 &\neq (y_1)^2 + (y_2)^2
\end{aligned}$$

Problem 3

For each of the following equations in parts (a)-(e),

- classify the equation as an ODE or a PDE
- state the order of the equation
- list the dependent and independent variables
- state whether the equation is linear or nonlinear
- if the equation is linear, state whether the equation has variable or constant coefficients and whether the equation is homogeneous or non-homogeneous

(a)

$$5\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} + u = 0$$

PDE, second order, dependent variables (u), independent variables (x, t), linear, constant coefficients, homogeneous

(b)

$$\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + 1\right) x = 10$$

ODE, first order, dependent variables (x), independent variables (t), non-linear

(c)

$$\sin(x)\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + \frac{\mathrm{d}y}{\mathrm{d}x} = -y$$

ODE, third order, dependent variables (y), independent variables (x), linear, variable coefficients, homogeneous

(d)

$$10y = (2 - y^3)\frac{\mathrm{d}y}{\mathrm{d}t} - \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$$

ODE, second order, dependent variables (y), independent variables (t), non-linear

(e)

$$2\theta + t^2 - \theta''' = 0$$

ODE, third order, dependent variables (θ), independent variables (t), linear, constant coefficients, non-homogeneous

Problem 4

Classify the following PDEs as linear or nonlinear, and as homogeneous or non-homogeneous.

(a)

$$u_{rr} - (\frac{\theta}{r})u_r = -\frac{u_\theta}{r^2}$$

linear and homogeneous

(b)

$$10 + 5\frac{\partial^2 u}{\partial x^2} + 25\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} = 0$$

linear and non-homogeneous

(c)

$$u_t = 4u_x x + 3uu_x$$

non-linear and non-homogeneous

Problem 5

Classify each of the following problems as an IVP, a BVP, or neither.

(a)

$$x^2 y'' + 7xy' + 3y = 5, \quad y(0) = 2, \quad y'(-1) = 0$$

boundary value problem

(b)

$$x^2 y'' + 7xy' + 3y = 5, \quad y(-1) = 2, \quad y'(-1) = 0$$

initial value problem

(c)

$$x^2y'' + 7xy' + 3y = 5, \quad y(0) = 2, \quad y(-1) = 0$$

boundary value problem

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech