

# Boundary Value Problems: Homework 1

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## Problem 1

Use the definition of a linear operator (given in Equation 1.1 in your textbook) to show that the operator  $L$  as defined  $Ly = 3Dy - 2y$  is a linear operator. Here  $D$  is the first-derivative operator.

$$\begin{aligned}L(c_1y_1 + c_2y_2) &= c_1Ly_1 + c_2Ly_2 \\3D(c_1y_1 + c_2y_2) - 2(c_1y_1 + c_2y_2) &= c_1(3Dy_1 - 2y_1) + c_2(3Dy_2 - 2y_2) \\3c_1Dy_1 + 3c_2Dy_2 - 2c_1y_1 - 2c_2y_2 &= 3c_1Dy_1 - 2c_1y_1 + 3c_2Dy_2 - 2c_2y_2\end{aligned}$$

## Problem 2

Use the definition of a linear operator (given in Equation 1.1 of your textbook) to show that the squaring operator,  $Q$ , defined as  $Qy = y^2$  is not linear.

$$\begin{aligned}L(y_1 + y_2) &= Ly_1 + Ly_2 \\Q(y_1 + y_2) &= Qy_1 + Qy_2 \\(y_1)^2 + 2y_1y_2 + (y_2)^2 &\neq (y_1)^2 + (y_2)^2\end{aligned}$$

## Problem 3

For each of the following equations in parts (a)-(e),

- classify the equation as an ODE or a PDE
- state the order of the equation
- list the dependent and independent variables
- state whether the equation is linear or nonlinear
- if the equation is linear, state whether the equation has variable or constant coefficients and whether the equation is homogeneous or non-homogeneous

(a)

$$5\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} + u = 0$$

PDE, second order, dependent variables ( $u$ ), independent variables ( $x, t$ ), linear, constant coefficients, homogeneous

(b)

$$\left( \left( \frac{dx}{dt} \right)^2 + 1 \right) x = 10$$

ODE, first order, dependent variables ( $x$ ), independent variables ( $t$ ), non-linear

(c)

$$\sin(x) \frac{d^3 y}{dx^3} + \frac{dy}{dx} = -y$$

ODE, third order, dependent variables ( $y$ ), independent variables ( $x$ ), linear, variable coefficients, homogeneous

(d)

$$10y = (2 - y^3) \frac{dy}{dt} - \frac{d^2 y}{dt^2}$$

ODE, second order, dependent variables ( $y$ ), independent variables ( $t$ ), non-linear

(e)

$$2\theta + t^2 - \theta''' = 0$$

ODE, third order, dependent variables ( $\theta$ ), independent variables ( $t$ ), linear, constant coefficients, non-homogeneous

## Problem 4

Classify the following PDEs as linear or nonlinear, and as homogeneous or non-homogeneous.

(a)

$$u_{rr} - \left( \frac{\theta}{r} \right) u_r = -\frac{u_\theta}{r^2}$$

linear and homogeneous

(b)

$$10 + 5 \frac{\partial^2 u}{\partial x^2} + 25 \frac{\partial^2 u}{\partial x \partial y} + 5 \frac{\partial^2 u}{\partial y^2} = 0$$

linear and non-homogeneous

(c)

$$u_t = 4u_x x + 3uu_x$$

non-linear and non-homogeneous

## Problem 5

Classify each of the following problems as an IVP, a BVP, or neither.

(a)

$$x^2 y'' + 7xy' + 3y = 5, \quad y(0) = 2, \quad y'(-1) = 0$$

boundary value problem

(b)

$$x^2 y'' + 7xy' + 3y = 5, \quad y(-1) = 2, \quad y'(-1) = 0$$

initial value problem

(c)

$$x^2y'' + 7xy' + 3y = 5, \quad y(0) = 2, \quad y(-1) = 0$$

boundary value problem

If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)