

Linear Algebra

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Review 4

1. "Vandermonde Matrix"

$$\begin{aligned} |V_n| &= \prod_{1 \leq i < j \leq n} (x_j - x_i) \\ V_n &= \begin{vmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^{n-1} \end{vmatrix} \\ &= \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & x_2 - x_1 & x_2(x_2 - x_1) & \dots & x_2^{n-2}(x_2 - x_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n - x_1 & x_n(x_n - x_1) & \dots & x_n^{n-2}(x_n - x_1) \end{vmatrix} \\ &= \begin{vmatrix} x_2 - x_1 & x_2(x_2 - x_1) & \dots & x_2^{n-2}(x_2 - x_1) \\ \vdots & \vdots & \vdots & \vdots \\ x_n - x_1 & x_n(x_n - x_1) & \dots & x_n^{n-2}(x_n - x_1) \end{vmatrix} \\ &= (x_2 - x_1)(x_3 - x_1) \dots (x_n - x_1) |V_{n-1}| \\ &= (x_2 - x_1)(x_3 - x_1) \dots (x_n - x_1) \prod_{1 \leq i < j \leq n-1} (x_j - x_i) \\ &= \prod_{1 \leq i < j \leq n} (x_j - x_i) \end{aligned}$$

2. Finding the kernel of D^n

$$D^2 = D \circ D$$

$$D^3 = D \circ D \circ D$$

$$D^n = D \circ D^{n-1}$$

3. This boils down to solving a differential equation.

4. $x^n - 1 = (x - 1)(\sum_{i=1}^{n-1} x^i)$

$$I - L^2 = (I - L)(I + L)$$

$$I = (I - L)(I + L)$$

$$I - L^n = (I - L) \left(\sum_{i=0}^{n-1} L^i \right)$$

Use this for part (a) and (c).

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech