

Linear Algebra

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Review 1

Exam 1 Point Values:

1. 8 points
2. 12 points
3. 8 points
4. 8 points
5. 10 points
6. 10 points
7. 12 points
8. 12 points
9. 4 points
10. 16 points

Total: 100 points

Exam 1 Topics:

1. A question about orthogonal vectors (refer to Section 1.2). If two vectors are orthogonal, their dot product is 0.

2. Refer to Exercises 66 and 67 from Section 1.2.
3. Prove a formula about vector length and dot products in \mathbb{R}^n . Look at 61-65 from Section 1.2.
4. Again, refer to exercises 61-65 in Section 1.2
5. True/False questions. Be able to recognize linear equations.
6. Given a certain system of linear equations, solve it. Example

$$\begin{aligned}x - y + z &= 0 \\-x + 3y + z &= 5 \\3x + y + 7z &= 2\end{aligned}$$

$$A' = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 3 & 1 & 5 \\ 3 & 1 & 7 & 2 \end{array} \right]$$

$$R_1 + R_2 \rightarrow R_2$$

$$-3R_1 + R_3 \rightarrow R_3$$

$$A' = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 2 & 5 \\ 0 & 4 & 4 & 2 \end{array} \right]$$

$$-2R_2 + R_3 \rightarrow R_3$$

$$A' = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 2 & 5 \\ 0 & 0 & 0 & -8 \end{array} \right]$$

$$0 \neq -8$$

In this example, this system is inconsistent and has no solution.

7. Let A be a 6×9 matrix and consider the homogeneous system $A\vec{x} = \vec{0}$ where \vec{x} is in \mathbb{R}^9 .
 - Is the system consistent?
 - Yes this system is consistent because $\vec{x} = \vec{0}$ solves it.

- What is the smallest number of free variables the system could have?

$$\text{free variables} = n - \text{rank}(A) = 9 - \text{rank}(A) \geq 9 - 6 = 3$$

by the Rank Theorem.

- Could the solution set for this system be a point, a line, or a plane?
No, because the dimension of these are not high enough.

8. Examine exercises 40-43 from Section 2.2.

9. Given 2 vectors \vec{u} and \vec{v} , compute the projection vector:

$$\text{proj}_{\vec{u}}\vec{v} = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\vec{u}\right)$$

10. Refer to question 39 from Section 2.2.

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech