Linear Algebra

Alvin Lin

August 2017 - December 2017

Review 1

Exam 1 Point Values:

1. 8 points
2. 12 points
3. 8 points
4. 8 points
5. 10 points
6. 10 points
7. 12 points
8. 12 points
9. 4 points
10. 16 points

Total: 100 points

Exam 1 Topics:

1. A question about orthogonal vectors (refer to Section 1.2). If two vectors are orthogonal, their dot product is 0.

2. Refer to Exercises 66 and 67 from Section 1.2.

3. Prove a formula about vector length and dot products in $\mathbb{R}^n$. Look at 61-65 from Section 1.2.

4. Again, refer to exercises 61-65 in Section 1.2

5. True/False questions. Be able to recognize linear equations.

6. Given a certain system of linear equations, solve it. Example

$$\begin{aligned} x - y + z &= 0 \\ -x + 3y + z &= 5 \\ 3x + y + 7z &= 2 \end{aligned}$$

$$A' = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 3 & 1 & 5 \\ 3 & 1 & 7 & 2 \end{array}\right]$$

$$R_1 + R_2 \to R_2$$
$$-3R_1 + R_3 \to R_3$$

$$A' = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 2 & 5 \\ 0 & 4 & 4 & 2 \end{array}\right]$$

$$-2R_2 + R_3 \to R_3$$

$$A' = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 2 & 5 \\ 0 & 0 & 0 & -8 \end{array}\right]$$

$$0 \neq -8$$

In this example, this system is inconsistent and has no solution.

7. Let A be a 6x9 matrix and consider the homogeneous system $A\vec{x} = \vec{0}$ where $\vec{x}$ is in $\mathbb{R}^n$.

   - Is the system consistent?
     Yes this system is consistent because $\vec{x} = \vec{0}$ solves it.

- What is the smallest number of free variables the system could have?

$$\text{free variables} = n - rank(A) = 9 - rank(A) \geq 9 - 6 = 3$$

  by the Rank Theorem.
- Could the solution set for this system be a point, a line, or a plane? No, because the dimension of these are not high enough.

8. Examine exercises 40-43 from Section 2.2.

9. Given 2 vectors $\vec{u}$ and $\vec{v}$, compute the projection vector:

$$proj_{\vec{u}}\vec{v} = (\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\vec{u})$$

10. Refer to question 39 from Section 2.2.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech