# Linear Algebra: Homework 2

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## Exercise 1

Find $\vec{u} \cdot \vec{v}$:

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\vec{u} \cdot \vec{v} = (-3) + 2 = -1$$

## Exercise 3

Find $\vec{u} \cdot \vec{v}$:

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\vec{u} \cdot \vec{v} = 2 + 6 + 3 = 11$$

## Exercise 5

Find $\vec{u} \cdot \vec{v}$:

$$\vec{u} = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{3} \\ 0 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 4 \\ -\sqrt{2} \\ 0 \\ -5 \end{bmatrix}$$

$$\vec{u} \cdot \vec{v} = 4 + (-2) + 0 + 0 = 2$$

## Exercise 7

Find $\|\vec{u}\|$ for the given exercise and give a unit vector in the direction of $\vec{u}$.

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\|\vec{u}\| = \sqrt{5}$$

$$\text{unit vector} = \langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$$

## Exercise 9

Find $\|\vec{u}\|$ for the given exercise and give a unit vector in the direction of $\vec{u}$.

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\|\vec{u}\| = \sqrt{14}$$

$$\text{unit vector} = \langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \rangle$$

## Exercise 11

Find $\|\vec{u}\|$ for the given exercise and give a unit vector in the direction of $\vec{u}$.

$$\vec{u} = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{3} \\ 0 \end{bmatrix}$$

$$\|\vec{u}\| = \sqrt{14}$$

$$\text{unit vector} = \langle \frac{1}{\sqrt{14}}, \sqrt{\frac{1}{7}}, \sqrt{\frac{3}{14}}, 0 \rangle$$

## Exercise 13

Find the distance $d(u, v)$ between $\vec{u}$ and $\vec{v}$ in the given exercise:

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$d(u, v) = \|\vec{u} - \vec{v}\| = \|\langle -4, 1 \rangle\| = \sqrt{17}$$

## Exercise 15

Find the distance $d(u, v)$ between $\vec{u}$ and $\vec{v}$ in the given exercise:

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$d(u, v) = \|\vec{u} - \vec{v}\| = \|\langle -1, -1, 2 \rangle\| = \sqrt{6}$$

## Exercise 19

Determine whether the angle between the vectors is acute, obtuse, or a right angle.

$$\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$
\begin{aligned}
\cos\theta &= \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} \\
&= \frac{2+2+(-1)}{\sqrt{6}\sqrt{6}} \\
&= \frac{3}{6} \\
&= \frac{1}{2} \\
\theta &= \cos^{-1}\frac{1}{2} \\
&= 60^\circ
\end{aligned}
$$

Acute.

## Exercise 23

Determine whether the angle between the vectors is acute, obtuse, or a right angle.

$$
\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}
$$

$$
\begin{aligned}
\cos\theta &= \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} \\
&= \frac{5+12+21+32}{\sqrt{30}\sqrt{174}} \\
&= \frac{70}{36\sqrt{145}} \\
&= \frac{35}{18\sqrt{145}} \\
\theta &\approx 80.707^\circ
\end{aligned}
$$

Acute.

## 25

Find the angle between $\vec{u}$ and $\vec{v}$:

$$
\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}
$$

$$
\theta = 60^\circ
$$

Work shown above.

## 31

Let A = (1,1,-1), B = (-3,2,-2), C = (2,2,-4). Prove that $\triangle$ ABC is a right-angled triangle.

$$\begin{aligned}\vec{AB} &= \langle -4, 1, -1\rangle \\ \vec{BC} &= \langle 5, 0, -2\rangle \\ \vec{AC} &= \langle 1, 1, -3\rangle \\ \vec{AB}\cdot\vec{BC} &= (-20) + 0 + 2 = -18 \\ \vec{AB}\cdot\vec{AC} &= (-4) + 1 + 3 = 0\end{aligned}$$

Since $\vec{AB}$ and $\vec{AC}$ have a dot product of 0, they must be orthogonal, meaning that the $\triangle$ ABC has a right angle.

## 41

Find the projection of $\vec{v}$ onto $\vec{u}$.

$$\vec{u} = \begin{bmatrix} \frac{3}{5} \\ \frac{-4}{5} \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{aligned}\text{proj}_{\vec{u}}\vec{v} &= \frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\vec{u} \\ &= \frac{\frac{3}{5} - \frac{8}{5}}{1+4}\vec{u} \\ &= \frac{-1}{5}\vec{u} \\ &= \langle \frac{-3}{25}, \frac{4}{25}\rangle\end{aligned}$$

## 43

Find the projection of $\vec{v}$ onto $\vec{u}$.

$$\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 2 \\ -3 \\ -1 \\ -2 \end{bmatrix}$$

$$\begin{aligned}\text{proj}_{\vec{u}}\vec{v} &= \frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\vec{u} \\ &= \frac{2+3+(-1)+2}{1+1+1+1}\vec{u} \\ &= \frac{6}{4}\vec{u} \\ &= \langle \frac{3}{2}, -\frac{3}{2}, \frac{3}{2}, -\frac{3}{2}\rangle\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech