

Linear Algebra: Homework 2

Alvin Lin

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Exercise 1

Find $\vec{u} \cdot \vec{v}$:

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$$\vec{u} \cdot \vec{v} = (-3) + 2 = -1$$

Exercise 3

Find $\vec{u} \cdot \vec{v}$:

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$
$$\vec{u} \cdot \vec{v} = 2 + 6 + 3 = 11$$

Exercise 5

Find $\vec{u} \cdot \vec{v}$:

$$\vec{u} = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{3} \\ 0 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 4 \\ -\sqrt{2} \\ 0 \\ -5 \end{bmatrix}$$
$$\vec{u} \cdot \vec{v} = 4 + (-2) + 0 + 0 = 2$$

Exercise 7

Find $\|\vec{u}\|$ for the given exercise and give a unit vector in the direction of \vec{u} .

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$
$$\|\vec{u}\| = \sqrt{5}$$
$$\text{unit vector} = \left\langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

Exercise 9

Find $\|\vec{u}\|$ for the given exercise and give a unit vector in the direction of \vec{u} .

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\|\vec{u}\| = \sqrt{14}$$

$$\text{unit vector} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

Exercise 11

Find $\|\vec{u}\|$ for the given exercise and give a unit vector in the direction of \vec{u} .

$$\vec{u} = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{3} \\ 0 \end{bmatrix}$$

$$\|\vec{u}\| = \sqrt{14}$$

$$\text{unit vector} = \left\langle \frac{1}{\sqrt{14}}, \sqrt{\frac{1}{7}}, \sqrt{\frac{3}{14}}, 0 \right\rangle$$

Exercise 13

Find the distance $d(u, v)$ between \vec{u} and \vec{v} in the given exercise:

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$d(u, v) = \|\vec{u} - \vec{v}\| = \|\langle -4, 1 \rangle\| = \sqrt{17}$$

Exercise 15

Find the distance $d(u, v)$ between \vec{u} and \vec{v} in the given exercise:

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$d(u, v) = \|\vec{u} - \vec{v}\| = \|\langle -1, -1, 2 \rangle\| = \sqrt{6}$$

Exercise 19

Determine whether the angle between the vectors is acute, obtuse, or a right angle.

$$\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$\begin{aligned}
\cos \theta &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} \\
&= \frac{2 + 2 + (-1)}{\sqrt{6}\sqrt{6}} \\
&= \frac{3}{6} \\
&= \frac{1}{2} \\
\theta &= \cos^{-1} \frac{1}{2} \\
&= 60^\circ
\end{aligned}$$

Acute.

Exercise 23

Determine whether the angle between the vectors is acute, obtuse, or a right angle.

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$\begin{aligned}
\cos \theta &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} \\
&= \frac{5 + 12 + 21 + 32}{\sqrt{30}\sqrt{174}} \\
&= \frac{70}{36\sqrt{145}} \\
&= \frac{35}{18\sqrt{145}} \\
\theta &\approx 80.707^\circ
\end{aligned}$$

Acute.

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Find the angle between \vec{u} and \vec{v} :

$$\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$\theta = 60^\circ$$

Work shown above.

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Let $A = (1,1,-1)$, $B = (-3,2,-2)$, $C = (2,2,-4)$. Prove that $\triangle ABC$ is a right-angled triangle.

$$\begin{aligned}\vec{AB} &= \langle -4, 1, -1 \rangle \\ \vec{BC} &= \langle 5, 0, -2 \rangle \\ \vec{AC} &= \langle 1, 1, -3 \rangle \\ \vec{AB} \cdot \vec{BC} &= (-20) + 0 + 2 = -18 \\ \vec{AB} \cdot \vec{AC} &= (-4) + 1 + 3 = 0\end{aligned}$$

Since \vec{AB} and \vec{AC} have a dot product of 0, they must be orthogonal, meaning that the $\triangle ABC$ has a right angle.

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Find the projection of \vec{v} onto \vec{u} .

$$\vec{u} = \begin{bmatrix} \frac{3}{5} \\ \frac{-4}{5} \\ \frac{4}{5} \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{aligned}proj_{\vec{u}}\vec{v} &= \frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\vec{u} \\ &= \frac{\frac{3}{5} - \frac{8}{5}}{1 + 4}\vec{u} \\ &= \frac{-1}{5}\vec{u} \\ &= \left\langle \frac{-3}{25}, \frac{4}{25} \right\rangle\end{aligned}$$

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Find the projection of \vec{v} onto \vec{u} .

$$\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 2 \\ -3 \\ -1 \\ -2 \end{bmatrix}$$

$$\begin{aligned}proj_{\vec{u}}\vec{v} &= \frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\vec{u} \\ &= \frac{2 + 3 + (-1) + 2}{1 + 1 + 1 + 1}\vec{u} \\ &= \frac{6}{4}\vec{u} \\ &= \left\langle \frac{3}{2}, -\frac{3}{2}, \frac{3}{2}, -\frac{3}{2} \right\rangle\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech