

Multivariable and Vector Calculus: Homework 10

Alvin Lin

August 2016 - December 2016

Section 16.2

Exercise 3

Evaluate the line integral, where C is the given curve.

$$\int_C xy^4 \, ds \quad C \text{ is the right half of the circle } x^2 + y^2 = 16$$

$$\begin{aligned} ds &= \sqrt{(-4 \sin(t))^2 + (4 \cos(t))^2} \, dt \\ &= 4 \, dt \end{aligned}$$

$$\begin{aligned} \int_C xy^4 \, ds &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4 \cos(t))(4 \sin(t))^4 (4 \, dt) \\ &= 4^6 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t) \sin^4(t) \, dt \\ &= 4^6 \left[\frac{\sin^5(t)}{5} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{4096(2)}{5} = \frac{8192}{5} \end{aligned}$$

Exercise 11

Evaluate the line integral, where C is the given curve.

$$\int_C xe^{yz} \, ds \quad C \text{ is the line segment from } (0, 0, 0) \text{ to } (1, 2, 3)$$

$$\begin{aligned}
c(t) &= \langle t, 2t, 3t \rangle \quad 0 \leq t \leq 1 \\
c'(t) &= \langle 1, 2, 3 \rangle \\
|c'(t)| &= \sqrt{14} \\
ds &= \sqrt{14} dt \\
\int_C xe^{yz} ds &= \int_0^1 te^{(2t)(3t)} \sqrt{14} dt \\
&= \sqrt{14} \int_0^1 te^{6t^2} dt \\
&= \sqrt{14} \left[\frac{e^{6x^2}}{12} \right]_0^1 \\
&= \sqrt{14} \frac{e^6 - 1}{12}
\end{aligned}$$

Exercise 21

Evaluate the line integral $\int_C F dr$, where C is given by the vector function $r(t)$.

$$F(x, y, z) = \sin(x)\hat{i} + \cos(y)\hat{j} + xz\hat{k} \quad r(t) = t^3\hat{i} - t^2\hat{j} + t\hat{k} \quad (0 \leq t \leq 1)$$

$$\begin{aligned}
r(t) &= t^3\hat{i} - t^2\hat{j} + t\hat{k} \\
r'(t) &= 3t^2\hat{i} - 2t\hat{j} + \hat{k} \\
dr &= (3t^2\hat{i} - 2t\hat{j} + \hat{k}) dt \\
F(x, y, z) &= \sin(x)\hat{i} + \cos(y)\hat{j} + xz\hat{k} \\
&= \sin(t^3)\hat{i} + \cos(-t^2)\hat{j} + t^3(t)\hat{k} \\
\int_C F \cdot dr &= \int_0^1 3t^2 \sin(t^3) - 2t \cos(t^2) + t^4 dt \\
&= \left[-\cos(t^3) - \sin(t^2) + \frac{t^5}{5} \right]_0^1 \\
&= \frac{6}{5} - \cos(1) - \sin(1)
\end{aligned}$$

Exercise 25

Use a calculator to evaluate the line integral correct to four decimal places.

$$\int_C xy \tan^{-1} z ds \quad \text{where } C \text{ has parametric equations } x = t^2, y = t^3, z = \sqrt{t}, 1 \leq t \leq 2$$

$$\begin{aligned}
r(t) &= \langle t^2, t^3, \sqrt{t} \rangle \\
r'(t) &= \langle 2t, 3t^2, \frac{1}{2\sqrt{t}} \rangle \\
|r'(t)| &= \sqrt{4t^2 + 9t^2 + \frac{1}{4t}} \\
\int_C xy \tan^{-1} z ds &= \int_1^2 (t^2)(t^3) \tan^{-1} \sqrt{t} \sqrt{4t^2 + 9t^2 + \frac{1}{4t}} dt \\
&\approx 94.8232
\end{aligned}$$

Exercise 32

Find the work done by the force field $F(x, y) = x^2\hat{i} + xy\hat{j}$ on a particle that moves once around the circle $x^2 + y^2 = 4$ oriented in the counter clockwise direction.

$$\begin{aligned}r(t) &= 2\cos(t)\hat{i} + 2\sin(t)\hat{j} \quad (0 \leq t \leq 2\pi) \\r'(t) &= -2\sin(t)\hat{i} + 2\cos(t)\hat{j} \\ \int_C F \cdot dr &= \int_0^{2\pi} 4\cos^2(t)(-2\sin(t)) + (2\cos(t))(2\sin(t))(2\cos(t)) dt \\ &= \int_0^{2\pi} -8\cos^2(t)\sin(t) + 8\cos^2(t)\sin(t) dt \\ &= \int_0^{2\pi} 0 dt \\ &= 0\end{aligned}$$

Exercise 41

Find the work done by the force field

$$F(x, y, z) = \langle x - y^2, y - z^2, z - x^2 \rangle$$

on a particle that moves along the line segment from $(0,0,1)$ to $(2,1,0)$.

$$\begin{aligned}r(t) &= \langle 2t, t, 1 - t \rangle \quad (0 \leq t \leq 1) \\r'(t) &= \langle 2, 1, -1 \rangle \\ F(x, y, z) &= \langle 2t - t^2, -t^2 + 3t - 1, 1 - t - 4t^2 \rangle \\ \int_0^1 F(x, y, z) \cdot r'(t) dt &= \int_0^1 2(2t - t^2) + (-t^2 - 3t - 1) - (1 - t - 4t^2) dt \\ &= \int_0^1 4t - 2t^2 - t^2 + 3t - 1 - 1 + t + 4t^2 dt \\ &= \int_0^1 t^2 + 8t - 2 dt \\ &= \left[\frac{t^3}{3} + \frac{8t^2}{2} - 2t \right]_0^1 \\ &= \frac{1}{3} + 4 - 2 \\ &= \frac{7}{3}\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech