

Multivariable and Vector Calculus

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Line Integral

$$\int_C f \, ds = \int_a^b f|v'(t)| \, dt$$
$$\int_C \vec{F} \, ds = \int_C (\vec{F} \cdot \vec{T}) \, ds = \int_a^b \vec{F} \cdot v'(t) \, dt$$

Find the work done against gravity by a 160 pound man going 3 revolutions up a silo $r = 20, h = 90$, losing 9 pounds out of 25 pounds of paint as it leaks from the bucket he is carrying up the silo.

$$r(t) = \left\langle 20 \cos(t), 20 \sin(t), \frac{15t}{\pi} \right\rangle$$
$$\vec{F} = \left\langle 0, 0, -185 + \frac{9t}{6\pi} \right\rangle$$
$$W = \int_C \vec{F} \, dr$$
$$= \int_0^{6\pi} \vec{F} \cdot r'(t) \, dt$$
$$= \int_0^{6\pi} -185 + \frac{9t}{6\pi} \, dt$$

Fundamental Theorem for Line Integrals

$$\int_{C_{AB}} \vec{\nabla} f \cdot dr = f(B) - f(A)$$

Note:

- If $\vec{F} = \vec{\nabla}f$, then F is a conservative vector field, and f is a potential function.
- If $\int_{C_{AB}^1} \vec{\nabla}f \cdot dr = \int_{C_{AB}^2} \vec{\nabla}f \cdot dr$ for all paths $A, B \in D$ and for all curves C_{AB} , then F is conservative in D .
- $\int_{C_{AA}} \vec{\nabla}f \cdot dr = 0$
- If $F = \langle P(x, y), Q(x, y) \rangle$ and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$, $\frac{\partial^2 P}{\partial y \partial x} = \frac{\partial^2 Q}{\partial x \partial y}$ are continuous, then F is conservative in D .
- D is open if and only if for every $(x_0, y_0) \in D$ there exist $\epsilon > 0$ such that the distance of radius ϵ and $(x_0, y_0) \in D$.
- D is connected if and only if for every $A, B \in D$ there exist $C_{AB} \subset D$.
- D is simply connected if and only if it is connected and every $C_{AA} \subset D$ has interior $C_{AA} \subset D$.

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech