Multivariable and Vector Calculus

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Linear Approximation

$$F(x, y, z) = 0$$

In a special case:

$$z - f(x, y) = 0$$

Then:

$$-f_x(x_\circ, y_\circ)(x - x_\circ) - f_y(x_\circ, y_\circ)(y - y_\circ) + 1(z - f(x_\circ, y_\circ)) = 0$$

$$\begin{aligned} z - f(x_\circ, y_\circ) &= f_x(x_\circ, y_\circ)(x - x_\circ) + f_y(x_\circ, y_\circ)(y - y_\circ) \\ f(x, y) - f(x_\circ, y_\circ) &= f_x(x_\circ, y_\circ)(x - x_\circ) + f_y(x_\circ, y_\circ)(y - y_\circ) \\ f(x, y) &\approx f(x_\circ, y_\circ) + f_x(x_\circ, y_\circ)(x - x_\circ) + f_y(x_\circ, y_\circ)(y - y_\circ) \end{aligned}$$

This is known as the linearization of the function. It provides a linear approximation of the function.

Example

Give the linear approximation of $f(x, y) = \sqrt{x^2 + y^2}$ at (4,3).

$$\begin{aligned} \sqrt{x^2 + y^2} &\approx 5 + (\frac{1(2x)}{2\sqrt{x^2 + y^2}})(x - 4) + \frac{1}{2\sqrt{x^2 + y^2}}(y - 3) \\ &\approx 5 + \frac{4}{5}(x - 4) + \frac{3}{5}(y - 3) \end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech