

# Multivariable and Vector Calculus

Alvin Lin

August 2017 - December 2017

## Linear Approximation

$$F(x, y, z) = 0$$

In a special case:

$$z - f(x, y) = 0$$

Then:

$$-f_x(x_0, y_0)(x - x_0) - f_y(x_0, y_0)(y - y_0) + 1(z - f(x_0, y_0)) = 0$$

$$z - f(x_0, y_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$f(x, y) - f(x_0, y_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

This is known as the linearization of the function. It provides a linear approximation of the function.

### Example

Give the linear approximation of  $f(x, y) = \sqrt{x^2 + y^2}$  at (4,3).

$$\begin{aligned}\sqrt{x^2 + y^2} &\approx 5 + \left(\frac{1(2x)}{2\sqrt{x^2 + y^2}}\right)(x - 4) + \frac{1}{2\sqrt{x^2 + y^2}}(y - 3) \\ &\approx 5 + \frac{4}{5}(x - 4) + \frac{3}{5}(y - 3)\end{aligned}$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)