

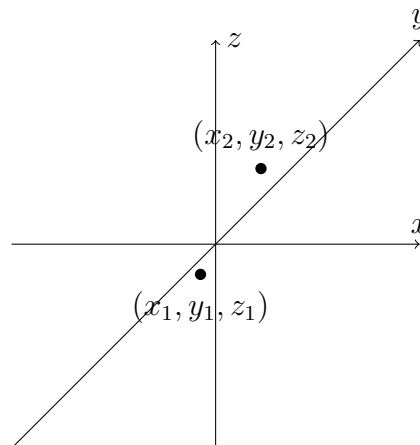
# Multivariable and Vector Calculus

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## Cartesian Coordinates

We take a standard x-y plane and put another axis perpendicular to it called the z-plane and plot points as  $(x, y, z)$ . This is known as the Cartesian coordinate system.



Cartesian Distance:

$$d((x_1, y_1, z_1), (x_2, y_2, z_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Sphere of radius  $r$ , center  $(x_o, y_o, z_o)$ :

$$(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2 = r^2$$

### Example

Give the largest sphere with center at  $(2, 7, 5)$  which fits within the I octant ( $x > 0, y > 0, z > 0$ ).

$$(x - 2)^2 + (y - 7)^2 + (z - 5)^2 = 2^2$$

## Example

$$x^2 + y^2 + z^2 - 4x + 2y - 6z = 1$$

What is the center and radius of such a circle?

$$\begin{aligned}x^2 - 4x + y^2 + 2y + z^2 - 6z &= 1 \\x^2 - 4x + y^2 + 2y + z^2 - 6z &= 1 \\[(x - 2)^2 - 4] + [(y + 1)^2 - 1] + [(z - 3)^2 - 9] &= 1 + 4 + 1 + 9 \\(x - 2)^2 + (y + 1)^2 + (z - 3)^2 &= 15\end{aligned}$$

## Example

Are the points  $P_1(1, 2, 3)$ ,  $P_2(2, 3, 4)$ ,  $P_3(0, 2, 6)$  colinear?

$$\text{dist}(P_1, P_2) = \sqrt{3}$$

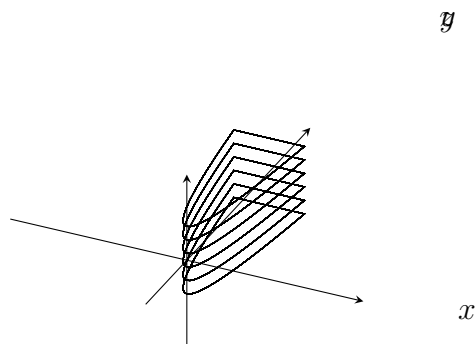
$$\text{dist}(P_2, P_3) = 3$$

$$\text{dist}(P_1, P_3) = \sqrt{10}$$

Two of the distances should sum up to the third. This is not true for any of these, thus the lines are not colinear.

## Graphing in 3D

Graph  $y = x^2$ :



This graph extends up and down the z axis infinitely and form a surface.

You can find all my notes at <http://omgimanagerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanagerd.tech](mailto:alvin@omgimanagerd.tech)