

# Homework #4

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## 1

Determine whether each of the following expressions is true or false.

- (a)  $x \in \{x\}$  True
- (b)  $\{x\} \in \{\{x\}\}$  True
- (c)  $\{x\} \subset \{x\}$  True
- (d)  $\emptyset \subset \{x\}$  True
- (e)  $\emptyset \in \{x\}$  False

## 2

Find the power set of each set below. Assume that  $a$  and  $b$  are distinct elements.

- (a)  $P(\{a\}) = \{\emptyset, \{a\}\}$
- (b)  $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- (c)  $P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

## 3

Let  $A = \{a, b, c\}$ ,  $B = \{c, d\}$  and  $C = \{x, z\}$ . Find

- (a)  $A \times C = \{(a, c), (a, d), (b, c), (b, d), (c, c), (c, d)\}$
- (b)  $C \times A \times B = \{(x, a, c), (x, a, d), (x, b, c), (x, b, d), (x, c, c), (x, c, d), (z, a, c), (z, a, d), (z, b, c), (z, b, d), (z, c, c), (z, c, d)\}$

$$(c) A \times B \times C = \{(a, c, x), (a, c, z), (a, d, x), (a, d, z), \\ (b, c, x), (b, c, z), (b, d, x), (b, d, z), (c, c, x), (c, c, z), (c, d, x), (c, d, z)\}$$

#### 4

Suppose that

$$A = \{a, b, c, d, e, f\} \text{ and } B = \{a, b, c, d, e, f, g, h, i, j, k\}$$

Find the following

$$(a) A \cup B = \{a, b, c, d, e, f, g, h, i, j, k\}$$

$$(b) A - B = \emptyset$$

$$(c) A \cap B = \{a, b, c, d, e, f\}$$

$$(d) B - A = \{g, h, i, j, k\}$$

#### 5

Suppose that  $A$ ,  $B$ , and  $C$  are arbitrary sets. Show the following and justify:

$$(a) (A \cup B) \subseteq (A \cup B \cup C)$$

$$(A \cup B) = \{x \mid x \in A \vee x \in B\}$$

$$(A \cup B \cup C) = \{x \mid x \in A \vee x \in B \vee x \in C\}$$

Identity Law

$$(b) A - B = A \cap \overline{B}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \cap \overline{B} = \{x \mid x \in A \wedge x \notin B\}$$

$$(c) (A - B) - C = (A - C) - (B - C)$$

$$(A - B) - C = \{x \mid x \in A \wedge x \notin B \wedge x \notin C\}$$

$$(A - C) = \{x \mid x \in A \wedge x \notin C\}$$

$$(B - C) = \{x \mid x \in B \wedge x \notin C\}$$

$$(A - C) - (B - C) = \{x \mid x \in A \wedge x \notin C\} - \{x \mid x \in B \wedge x \notin C\}$$

$$(A - C) - (B - C) = \{x \mid x \in A \wedge x \notin B \wedge x \notin C\}$$

$$(d) \overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$$

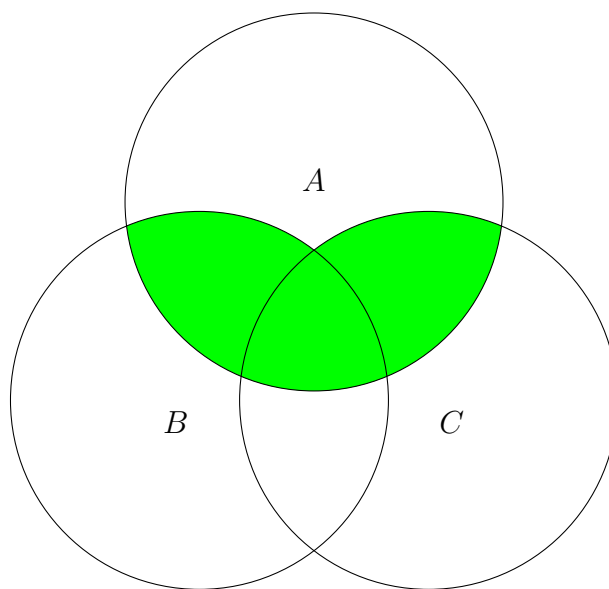
$$\begin{aligned} \overline{A \cup (B \cap C)} &= \overline{A} \cap \overline{B \cap C} \\ &= \overline{A} \cap (\overline{B} \cup \overline{C}) \\ &= (\overline{B} \cup \overline{C}) \cap \overline{A} \\ &= (\overline{C} \cup \overline{B}) \cap \overline{A} \end{aligned}$$

## 6

Suppose that  $A$ ,  $B$ , and  $C$  are sets. Draw the Venn diagrams for each of the combinations below.

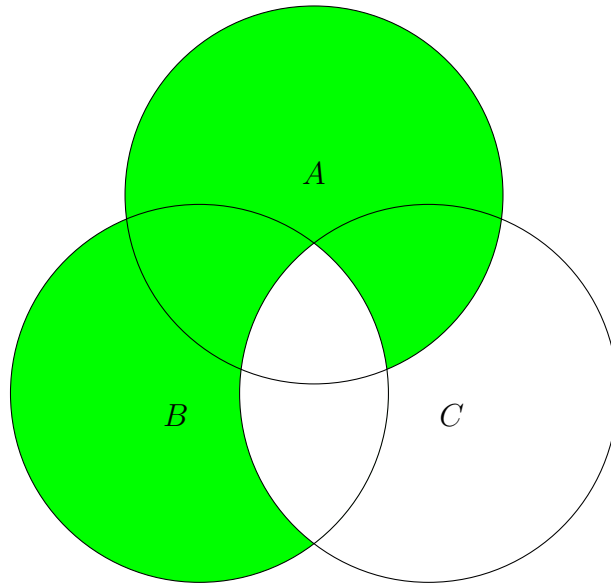
(a)

$$A \cap (B \cup C)$$



(b)

$$(A - B) \cup (A - C) \cup (B - C)$$



If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)