

Rules of Inference

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Rules of Inference

Proofs in mathematics are valid arguments that establish the truth of mathematical statements.

Argument: Sequence of statements that ends with a conclusion.

A conclusion is **valid** if it follows from the truth of the preceding statements (called premises). Rules of inference are basic tools for establishing the truth of statements.

Example

Consider the following:

- “If you have a current password, then you can log on to the network”
- “You have a current password”
- “Therefore, you can log onto the network”

Determine whether this is a valid argument. Does the conclusion follow? Let:

- p : you have a current password
- q : you can log onto the network

$$\begin{array}{l} p \rightarrow q \\ p \\ \therefore q \end{array}$$

$(p \leftrightarrow q \wedge p) \rightarrow q$ is a tautology, so the argument is valid.

Example

Suppose we replace p and q by:

- p : you have access to the network
- q : you can change your grade

$$p : \textit{True}$$

$$p \rightarrow q : \textit{False}$$

$p \rightarrow q$ is valid still, but since the premise $p \rightarrow q$ is false, you can't conclude q . $(p \wedge (p \rightarrow q)) \rightarrow q$ is a rule of inference called Modus Ponens. It is always valid, but the conclusion might not always be true.

Rules of Inference

1. Modus Ponens

$$p$$

$$p \rightarrow q$$

$$\therefore q$$

2. Modus Tollens

$$\neg q$$

$$p \rightarrow q$$

$$\therefore \neg p$$

3. Hypothetical Syllogism

$$p \rightarrow q$$

$$q \rightarrow r$$

$$p \rightarrow r$$

4. Disjunctive Syllogism

$$p \vee q$$

$$\neg p$$

$$\therefore q$$

5. Addition

$$p \\ \therefore p \vee q$$

6. Simplification

$$p \wedge q \\ \therefore p$$

7. Conjunction

$$p \\ q \\ \therefore p \wedge q$$

8. Resolution

$$p \vee q \\ \neg p \vee r \\ q \vee r$$

Example

Determine whether the argument is valid and whether its conclusion must be true. If $\sqrt{2} > \frac{3}{2}$ then $(\sqrt{2})^2 > (\frac{3}{2})^2$. Let:

$$p : \sqrt{2} > \frac{3}{2}$$

$$q : (\sqrt{2})^2 > (\frac{3}{2})^2$$

The conclusion is not true since one premise, namely $\sqrt{2} > \frac{3}{2}$ is false.

Example

What rules are used?

1. It is sunny now. Therefore, it is either sunny or 32°F .

Addition Rule

2. It is sunny and above 32°F now. Therefore, it is sunny now.

Simplification Rule

3. If it rains today, we will not have a bbq. If we do not have a bbq, then we will tomorrow.

Hypothetical Syllogism

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech