Section 10.4

Alvin Lin

Calculus II: August 2016 - December 2016

Exercise 5

$$r^2 = \sin(2\theta)$$

$$\begin{aligned}
A &= \frac{1}{2}\int_a^b r^2 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_0^{\pi/2} \sin(2\theta) \ \mathrm{d}\theta \\
&= \frac{1}{2}\left[\frac{-\cos(2\theta)}{2}\right]_0^{\pi/2} \\
&= \frac{1}{2}
\end{aligned}$$

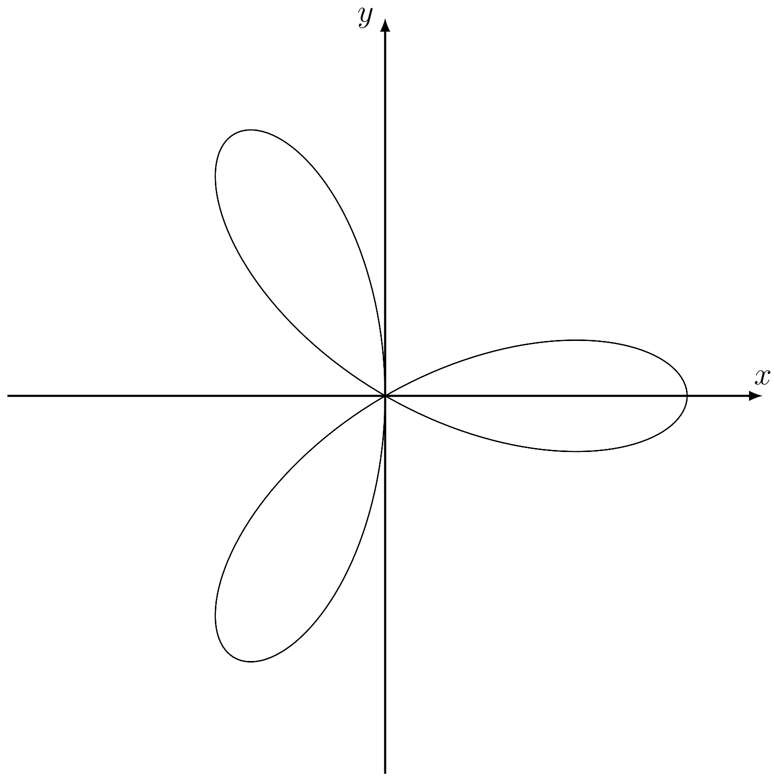
Exercise 7

$$r = 4 + 3\sin(\theta)$$

$$\begin{aligned}
A &= \frac{1}{2}\int_a^b r^2 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_{-\pi/2}^{\pi/2} (4+3\sin(\theta))^2 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_{-\pi/2}^{\pi/2} 9\sin^2(\theta) + 24\sin(\theta) + 16 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{9(1-\cos(2\theta))}{2} + 24\sin(\theta) + 16 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{-\cos(2\theta)}{2} + 24\sin(\theta) + \frac{41}{2} \ \mathrm{d}\theta \\
&= \frac{1}{2}\left[\frac{-9\sin(2\theta)}{4} - 24\cos(\theta) + \frac{41}{2}\theta\right]_{-\pi/2}^{\pi/2} \qquad = \frac{41\pi}{2}
\end{aligned}$$

Exercise 17

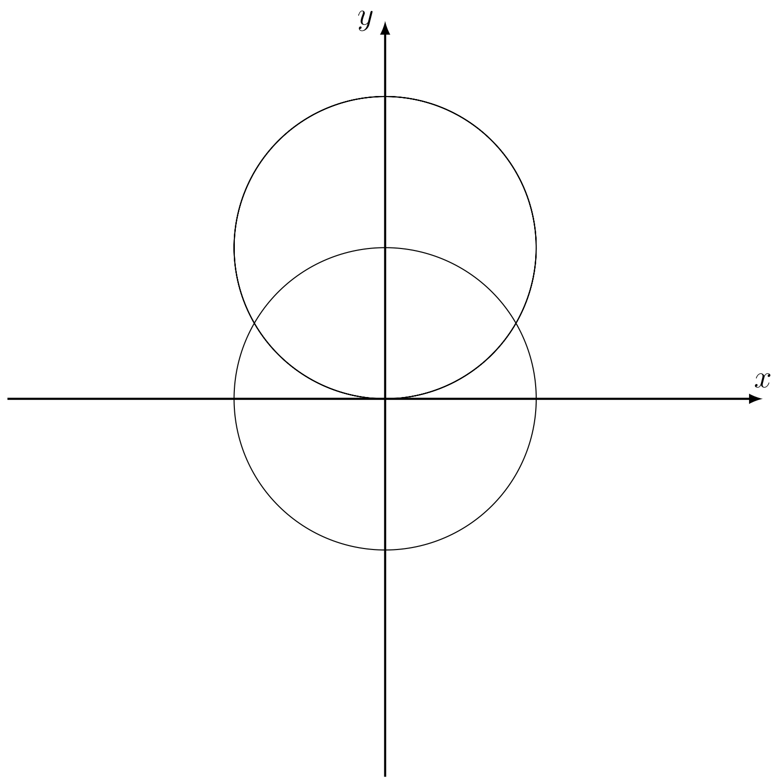
$$r = 4\cos(3\theta)$$



$$\begin{aligned}
A &= \frac{1}{2}\int_a^b r^2 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_{-\pi/6}^{\pi/6} (4\cos(3\theta))^2 \ \mathrm{d}\theta \\
&= 8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta) \ \mathrm{d}\theta \\
&= 8\int_{-\pi/6}^{\pi/6} \frac{1+\cos(6\theta)}{2} \ \mathrm{d}\theta \\
&= 4\int_{-\pi/6}^{\pi/6} 1+\cos(6\theta) \ \mathrm{d}\theta \\
&= 4\Big[\theta + \frac{\sin(6\theta)}{6}]_{-\pi/6}^{\pi/6} \\
&= \frac{4\pi}{3}
\end{aligned}$$

Exercise 23

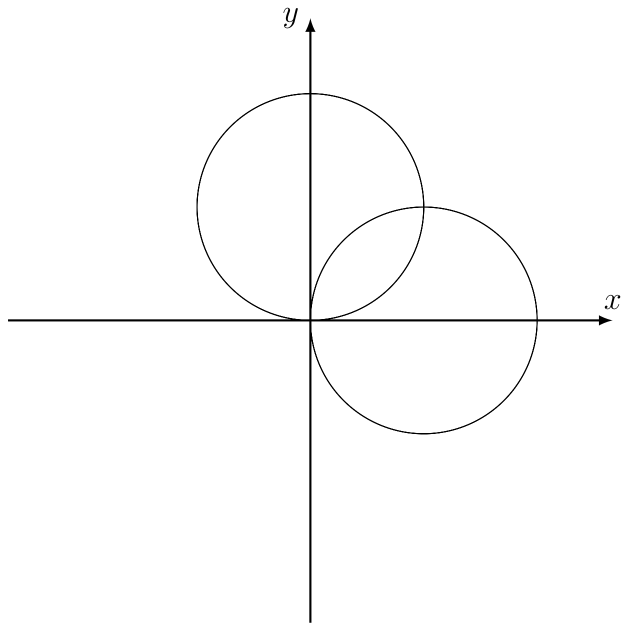
$$r_1 = 2 \quad r_2 = 4\sin(\theta)$$



$$
\begin{aligned}
A &= \frac{1}{2}\int_a^b (r_2)^2 - (r_1)^2 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4\sin(\theta))^2 - 2^2 \ \mathrm{d}\theta \\
&= \frac{1}{2}\int_{\pi/6}^{5\pi/6} 16\sin^2(\theta) - 4 \ \mathrm{d}\theta \\
&= 2\int_{\pi/6}^{5\pi/6} 4\sin^2(\theta) - 1 \ \mathrm{d}\theta \\
&= 2\int_{\pi/6}^{5\pi/6} 4\sin^2(\theta) - 2 + 1 \ \mathrm{d}\theta \\
&= 2\int_{\pi/6}^{5\pi/6} 2(2\sin^2(\theta) - 1) + 1 \ \mathrm{d}\theta \\
&= 2\int_{\pi/6}^{5\pi/6} -2\cos(2\theta) + 1 \ \mathrm{d}\theta \\
&= 2\Big[-\sin(2\theta) + \theta \Big]_{\pi/6}^{5\pi/5} \\
&= 2\bigg(\big[-\sin(\frac{5\pi}{3}) + \frac{5\pi}{6} \big] - \big[-\sin(\frac{\pi}{3}) + \frac{\pi}{6} \big] \bigg) \\
&= 2\sqrt{3} + \frac{4\pi}{3}
\end{aligned}
$$

Exercise 29

$$r = 3\cos(\theta) \quad r = 3\sin(\theta)$$



$$\begin{aligned}
A &= \frac{1}{2}\int_a^b r^2 \ \mathrm{d}\theta \\
&= \frac{1}{2}(2)\int_0^{\pi/4} (3\sin(\theta))^2 \ \mathrm{d}\theta \\
&= \int_0^{\pi/4} 9\sin^2(\theta) \ \mathrm{d}\theta \\
&= \int_0^{\pi/4} 9(\frac{1-\cos(2\theta)}{2}) \ \mathrm{d}\theta \\
&= \frac{9}{2}\int_0^{\pi/4} 1-\cos(2\theta) \ \mathrm{d}\theta \\
&= \frac{9}{2}\left[\theta - \frac{\sin(2\theta)}{2}\right]_0^{\pi/4} \\
&= \frac{9}{2}\left([\frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2}] - [0 - \frac{\sin(0)}{2}]\right) \\
&= \frac{9\pi}{8} - \frac{9}{4}
\end{aligned}$$

Exercise 45

$$r = 2\cos(\theta) \quad 0 \leq \theta \leq \pi$$

$$
\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}\theta} &= -2\sin(\theta) \\
L &= \int_a^b \sqrt{r^2 + (\frac{\mathrm{d}r}{\mathrm{d}\theta})^2}\ \mathrm{d}\theta \\
&= \int_0^\pi \sqrt{(2\cos(\theta))^2 + (-2\sin(\theta))^2}\ \mathrm{d}\theta \\
&= \int_0^\pi \sqrt{4\cos^2(\theta) + 4\sin^2(\theta)}\ \mathrm{d}\theta \\
&= \int_0^\pi \sqrt{4(\cos^2(\theta) + \sin^2(\theta))}\ \mathrm{d}\theta \\
&= \int_0^\pi \sqrt{4}\ \mathrm{d}\theta \\
&= \int_0^\pi 2\ \mathrm{d}\theta \\
&= \Big[2\theta\Big]_0^\pi \\
&= 2\pi
\end{aligned}
$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech