

Section 8.2

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Exercise 6

$$y = \tan^{-1}(x) \quad 0 \leq x \leq 2$$
$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Rotated about the x-axis:

$$S = \int (2\pi y) dS$$
$$= 2\pi \int_0^2 y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$
$$= 2\pi \int_0^2 \tan^{-1}(x) \sqrt{1 + \left(\frac{1}{1+x^2}\right)^2} dx$$
$$\approx 9.79564$$

Rotated about the y-axis:

$$S = 2\pi \int_0^2 x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$
$$\approx 13.721$$

Exercise 12

$$y = \frac{x^3}{6} + \frac{1}{2x} \quad \frac{1}{2} \leq x \leq 1$$
$$y = \frac{x^3}{6} + \frac{1}{2x} = \frac{x^4 + 3}{6x}$$
$$\frac{dy}{dx} = \frac{x^4 - 1}{2x^2}$$

Rotated about the x-axis:

$$\begin{aligned} S &= \int (2\pi y) \, dS \\ &= 2\pi \int_{\frac{1}{2}}^1 y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^1 \left(\frac{x^4 + 3}{6x}\right) \sqrt{1 + \left(\frac{x^4 - 1}{2x^2}\right)^2} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^1 \left(\frac{x^4 + 3}{6x}\right) \sqrt{1 + \frac{x^8 - 2x^4 + 1}{4x^4}} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^1 \left(\frac{x^4 + 3}{6x}\right) \sqrt{\frac{x^8 - 2x^4 + 4x^4 + 1}{4x^4}} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^1 \left(\frac{x^4 + 3}{6x}\right) \sqrt{\frac{x^8 + 2x^4 + 1}{4x^4}} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^1 \left(\frac{x^4 + 3}{6x}\right) \sqrt{\left(\frac{x^4 + 1}{2x^2}\right)^2} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^1 \left(\frac{x^4 + 3}{6x}\right) \frac{x^4 + 1}{2x^2} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^2 \frac{x^8 + 4x^4 + 3}{12x^3} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^2 \frac{x^8}{12x^3} + \frac{4x^4}{12x^3} + \frac{3}{12x^3} \, dx \\ &= 2\pi \int_{\frac{1}{2}}^2 \frac{x^5}{12} + \frac{x}{3} + \frac{1}{4x^3} \, dx \\ &= 2\pi \left[\frac{x^6}{72} + \frac{x^2}{6} - \frac{1}{8x^2} \right]_{\frac{1}{2}}^1 \\ &= \frac{263\pi}{256} \end{aligned}$$

Exercise 14

$$\begin{aligned} x &= 1 + 2y^2 \quad 1 \leq y \leq 2 \\ \frac{dx}{dy} &= 4y \end{aligned}$$

Rotated about the x-axis:

$$\begin{aligned} S &= \int (2\pi y) \, dS \\ &= 2\pi \int_1^2 y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy \\ &= 2\pi \int_1^2 y \sqrt{1 + (4y)^2} \, dy \\ &= 2\pi \int_1^2 y \sqrt{1 + 16y^2} \, dy \\ \text{Let : } u &= 1 + 16y^2 \quad du = 32y \, dy \\ &= 2\pi \int y \sqrt{u} \frac{du}{32y} \\ &= \frac{\pi}{16} \int \sqrt{u} \, du \\ &= \frac{\pi}{16} \frac{2u^{\frac{3}{2}}}{\frac{3}{2}} \\ &= \frac{\pi}{24} \left[(1 + 16y^2)^{\frac{3}{2}} \right]_{\frac{1}{2}}^1 \\ &\approx 59.42 \end{aligned}$$

Exercise 17

$$\begin{aligned} x &= \sqrt{a^2 - y^2} \quad 0 \leq y \leq \frac{a}{2} \\ \frac{dx}{dy} &= \frac{1}{2}(a^2 - y^2)^{-\frac{1}{2}}(-2y) = \frac{-y}{\sqrt{a^2 - y^2}} \end{aligned}$$

Rotated about the y-axis:

$$\begin{aligned} S &= \int 2\pi x \, dS \\ &= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dx \\ &= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2} \sqrt{1 + \left(\frac{-y}{\sqrt{a^2 - y^2}}\right)^2} \, dx \\ &= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2} \sqrt{1 + \frac{y^2}{a^2 - y^2}} \, dx \\ &= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2} \sqrt{\frac{a^2 - y^2 + y^2}{a^2 - y^2}} \, dx \\ &= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2} \sqrt{\frac{a^2}{a^2 - y^2}} \, dx \\ &= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2} \frac{\sqrt{a^2}}{\sqrt{a^2 - y^2}} \, dy \\ &= 2\pi \int_0^{a/2} 2\pi a \, dy \\ &= 2\pi a \left[y \right]_0^{a/2} \\ &= 2\pi a \left(\frac{a}{2} \right) \\ &= \pi a^2 \end{aligned}$$

Exercise 19

$$y = \frac{x^5}{5} \quad 0 \leq x \leq 5$$
$$\frac{dx}{dy} = x^4$$

Rotated about the x-axis:

$$\begin{aligned} S &= \int 2\pi y \, dS \\ &= 2\pi \int_0^5 \frac{x^5}{5} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dx \\ &= 2\pi \int_0^5 \frac{x^5}{5} \sqrt{1 + (x^4)^2} \, dx \\ &= \frac{2\pi}{5} \int_0^5 x^5 \sqrt{1 + x^8} \, dx \end{aligned}$$

Using Simpson's Rule with $n = 10$:

$$S \approx \frac{2\pi}{5} \times 979206.52 \approx 1230506.16$$

Exercise 22

$$y = x \ln(x) \quad 1 \leq x \leq 2$$

$$\frac{dy}{dx} = 1 + \ln(x)$$

Rotated about the x-axis:

$$\begin{aligned} S &= \int 2\pi y \, dS \\ &= 2\pi \int_0^2 x \ln(x) \sqrt{1 + (1 + \ln(x))^2} \, dx \\ &= 2\pi \int_0^2 x \ln(x) \sqrt{1 + 1 + 2\ln(x) + (\ln(x))^2} \, dx \\ &= 2\pi \int_0^2 x \ln(x) \sqrt{(\ln(x))^2 + 2\ln(x) + 2} \, dx \end{aligned}$$

Using Simpson's Rule with $n = 10$:

$$S \approx 7.25$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech