Section 8.2

Alvin Lin

Calculus II: August 2016 - December 2016

Section 8.2

Exercise 6

$$y = \tan^{-1}(x) \qquad 0 \leq x \leq 2$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x^2}$$

Rotated about the x-axis:

$$\begin{aligned} S &= \int (2\pi y)\ \mathrm{d}S \\ &= 2\pi \int_0^2 y\sqrt{1+(\frac{\mathrm{d}y}{\mathrm{d}x})^2}\ \mathrm{d}x \\ &= 2\pi \int_0^2 \tan^{-1}(x)\sqrt{1+(\frac{1}{1+x^2})^2}\ \mathrm{d}x \\ &\approx 9.79564 \end{aligned}$$

Rotated about the y-axis:

$$\begin{aligned} S &= 2\pi \int_0^2 x\sqrt{1+(\frac{\mathrm{d}y}{\mathrm{d}x})^2}\ \mathrm{d}x \\ &\approx 13.721 \end{aligned}$$

Exercise 12

$$y = \frac{x^3}{6} + \frac{1}{2x} \qquad \frac{1}{2} \leq x \leq 1$$
$$y = \frac{x^3}{6} + \frac{1}{2x} = \frac{x^4+3}{6x}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^4-1}{2x^2}$$

Rotated about the x-axis:

$$
\begin{aligned}
S &= \int (2\pi y)\ \mathrm{d}S \\
&= 2\pi \int_{\frac{1}{2}}^{1} y\sqrt{1 + (\frac{\mathrm{d}y}{\mathrm{d}x})^2}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{1} (\frac{x^4+3}{6x})\sqrt{1 + (\frac{x^4-1}{2x^2})^2}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{1} (\frac{x^4+3}{6x})\sqrt{1 + \frac{x^8-2x^4+1}{4x^4}}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{1} (\frac{x^4+3}{6x})\sqrt{\frac{x^8-2x^4+4x^4+1}{4x^4}}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{1} (\frac{x^4+3}{6x})\sqrt{\frac{x^8+2x^4+1}{4x^4}}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{1} (\frac{x^4+3}{6x})\sqrt{\left(\frac{x^4+1}{2x^2}\right)^2\ \mathrm{d}x} \\
&= 2\pi \int_{\frac{1}{2}}^{1} (\frac{x^4+3}{6x})\frac{x^4+1}{2x^2}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{2} \frac{x^8+4x^4+3}{12x^3}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{2} \frac{x^8}{12x^3} + \frac{4x^4}{12x^3} + \frac{3}{12x^3}\ \mathrm{d}x \\
&= 2\pi \int_{\frac{1}{2}}^{2} \frac{x^5}{12} + \frac{x}{3} + \frac{1}{4x^3}\ \mathrm{d}x \\
&= 2\pi \left[\frac{x^6}{72} + \frac{x^2}{6} - \frac{1}{8x^2}\right]_{\frac{1}{2}}^{1} \\
&= \frac{263\pi}{256}
\end{aligned}
$$

Exercise 14

$$
x = 1 + 2y^2 \quad 1 \leq y \leq 2
$$

$$
\frac{\mathrm{d}x}{\mathrm{d}y} = 4y
$$

Rotated about the x-axis:

$$
\begin{aligned}
S &= \int (2\pi y)\ \mathrm{d}S \\
&= 2\pi \int_1^2 y\sqrt{1+(\frac{\mathrm{d}x}{\mathrm{d}y})^2}\ \mathrm{d}x \\
&= 2\pi \int_1^2 y\sqrt{1+(4y)^2}\ \mathrm{d}y \\
&= 2\pi \int_1^2 y\sqrt{1+16y^2}\ \mathrm{d}y \\
&\mathit{Let:} \quad u = 1+16y^2 \quad \mathrm{d}u = 32y\ \mathrm{d}y \\
&= 2\pi \int y\sqrt{u}\frac{\mathrm{d}u}{32y} \\
&= \frac{\pi}{16}\int \sqrt{u}\ \mathrm{d}u \\
&= \frac{\pi}{16}\frac{2u^{\frac{3}{2}}}{3} \\
&= \frac{\pi}{24}\left[(1+8y^2)^{\frac{3}{2}}\right]_{\frac{1}{2}}^{1} \\
&\approx 59.42
\end{aligned}
$$

Exercise 17

$$x = \sqrt{a^2-y^2} \quad 0 \le y \le \frac{a}{2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{2}(a^2-y^2)^{-\frac{1}{2}}(-2y) = \frac{-y}{\sqrt{a^2-y^2}}$$

Rotated about the y-axis:

$$
\begin{aligned}
S &= \int 2\pi x \ \mathrm{d}S \\
&= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2}\sqrt{1 + (\frac{\mathrm{d}x}{\mathrm{d}y})^2} \ \mathrm{d}x \\
&= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2}\sqrt{1 + (\frac{-y}{\sqrt{a^2 - y^2}})^2} \ \mathrm{d}x \\
&= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2}\sqrt{1 + \frac{y^2}{a^2 - y^2}} \ \mathrm{d}x \\
&= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2}\sqrt{\frac{a^2 - y^2 + y^2}{a^2 - y^2}} \ \mathrm{d}x \\
&= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2}\sqrt{\frac{a^2}{a^2 - y^2}} \ \mathrm{d}x \\
&= 2\pi \int_0^{a/2} \sqrt{a^2 - y^2}\frac{\sqrt{a^2}}{\sqrt{a^2 - y^2}} \ \mathrm{d}y \\
&= 2\pi \int_0^{a/2} 2\pi a \ \mathrm{d}y \\
&= 2\pi a \Big[y \Big]_0^{a/2} \\
&= 2\pi a (\frac{a}{2}) \\
&= \pi a^2
\end{aligned}
$$

Exercise 19

$$
y = \frac{x^5}{5} \qquad 0 \leq x \leq 5
$$

$$
\frac{\mathrm{d}x}{\mathrm{d}y} = x^4
$$

Rotated about the x-axis:

$$
\begin{aligned}
S &= \int 2\pi y \ \mathrm{d}S \\
&= 2\pi \int_0^5 \frac{x^5}{5} \sqrt{1 + (\frac{\mathrm{d}x}{\mathrm{d}y})^2} \ \mathrm{d}x \\
&= 2\pi \int_0^5 \frac{x^5}{5} \sqrt{1 + (x^4)^2} \ \mathrm{d}x \\
&= \frac{2\pi}{5} \int_0^5 x^5 \sqrt{1 + x^8} \ \mathrm{d}x
\end{aligned}
$$

Using Simpson's Rule with n = 10:

$$S \approx \frac{2\pi}{5} \times 979206.52 \approx 1230506.16$$

Exercise 22

$$y = x \ln(x) \qquad 1 \leq x \leq 2$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln(x)$$

Rotated about the x-axis:

$$
\begin{aligned}
S &= \int 2\pi y \ \mathrm{d}S \\
&= 2\pi \int_0^2 x \ln(x) \sqrt{1 + (1 + \ln(x))^2} \ \mathrm{d}x \\
&= 2\pi \int_0^2 x \ln(x) \sqrt{1 + 1 + 2\ln(x) + (\ln(x))^2} \ \mathrm{d}x \\
&= 2\pi \int_0^2 x \ln(x) \sqrt{(\ln(x))^2 + 2\ln(x) + 2} \ \mathrm{d}x
\end{aligned}
$$

Using Simpson's Rule with n = 10:

$$S \approx 7.25$$

If you have any questions, comments, or concerns, please contact me at
alvin@omgimanerd.tech