Section 7.8

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Exercise 5

$$\int_3^\infty \frac{1}{(x-2)^{3/2}}\,\mathrm{d}x$$

$$\lim_{b\to\infty}\int_3^b (x-2)^{\frac{3}{2}}\,\mathrm{d}x$$

$$Let: u = x - 2$$

$$\mathrm{d}u = \mathrm{d}x$$

$$\lim_{b\to\infty}\int_3^b u^{\frac{3}{2}}\,\mathrm{d}u$$

$$\lim_{b\to\infty}\left[\frac{-1}{2\sqrt{u}}\right]_3^b$$

$$\lim_{b\to\infty}\left[\frac{-1}{2\sqrt{b}} - \frac{-1}{2\sqrt{3}}\right]$$

$$= \frac{\sqrt{3}}{6}$$

Exercise 7

$$\int_{-\infty}^0 \frac{1}{3-4x}\,\mathrm{d}x$$

$$\lim_{b\to-\infty}\int_b^0 \frac{1}{3-4x}\,\mathrm{d}x$$

$$Let: u = 3 - 4x$$

$$\mathrm{d}u = -4\,\mathrm{d}x$$

$$\frac{-1}{4}\lim_{b\to-\infty}\int_b^0 \frac{1}{u}\,\mathrm{d}u$$

$$\frac{-1}{4}\lim_{b\to-\infty}\left[\ln|3-4x|\right]_b^0$$

$$\frac{-1}{4}\lim_{b\to-\infty}\Big[\ln|3|-\ln|3-4b|\Big]$$
$$=\infty$$

Exercise 13

$$\int_{-\infty}^{\infty} x\mathrm{e}^{-x^2}\ \mathrm{d}x$$
$$\int_{-\infty}^{0} x\mathrm{e}^{-x^2}\ \mathrm{d}x+\int_{0}^{\infty} x\mathrm{e}^{-x^2}\ \mathrm{d}x$$

Solve as an indefinite integral:

$$\int x\mathrm{e}^{-x^2}\ \mathrm{d}x$$
$$\frac{-1}{2}\int -2x\mathrm{e}^{-x^2}\ \mathrm{d}x$$
$$e^{-x^2}+C$$

Substitute the original limits:

$$-\frac{1}{2}\lim_{a\to-\infty}\Big[\mathrm{e}^{-x^2}\Big]_a^0-\frac{1}{2}\lim_{b\to\infty}\Big[\mathrm{e}^{-x^2}\Big]_0^b$$
$$-\frac{1}{2}\lim_{a\to-\infty}\Big[\mathrm{e}^{-a^2}-\mathrm{e}^0\Big]-\frac{1}{2}\lim_{b\to\infty}\Big[\mathrm{e}^0-\mathrm{e}^{-b^2}\Big]$$
$$=0$$

Exercise 29

$$\int_{-2}^{14}(x+2)^{-1/4}\ \mathrm{d}x$$

Solve as an indefinite integral:

$$\int (x+2)^{-1/4}\ \mathrm{d}x$$
$$Let: f'(x)=x^{-1/4}\ \mathrm{d}x \quad g(x)=x+2$$
$$f(x)=\frac{4}{3}x^{3/4} \quad g'(x)=\ \mathrm{d}x$$
$$f(g(x))=\frac{4}{3}(x+2)^{3/4}$$

Substitute the original limits:

$$\frac{4}{3}\Big[(x+2)^{3/4}\Big]_{-2}^{14}$$

$$\frac{4}{3}\left[(14+2)^{3/4} - (-2+2)^{3/4}\right]$$

$$\frac{4}{3}\left[(14+2)^{3/4} - (-2+2)^{3/4}\right]$$

$$\frac{4}{3}\left[8-0\right]$$

$$= \frac{32}{3}$$

Exercise 33

$$\int_0^9 \frac{1}{\sqrt[3]{x-1}}\ \mathrm{d}x$$

Solve as an indefinite integral:

$$\int \frac{1}{\sqrt[3]{x-1}}\ \mathrm{d}x$$

$$Let: u = x - 1$$

$$\mathrm{d}u = \ \mathrm{d}x$$

$$\int u^{-2/3}\ \mathrm{d}u$$

$$3u^{1/3} + C$$

$$3(x-1)^{1/3} + C$$

Substitute the original limits:

$$3\left[(x-1)^{1/3}\right]_0^9$$

$$3\left[\sqrt[3]{0} - \sqrt[3]{8}]\right.$$

$$= -6$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech