Section 7.1

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Calculus II: August 2016 - December 2016

Section 5.5

Exercise 3

$$\int x\cos(5x)\ \mathrm{d}x$$

$$Let : u = x \qquad \mathrm{d}v = \cos(5x)\ \mathrm{d}x$$

$$\mathrm{d}u = \ \mathrm{d}x \quad v = \frac{\sin(5x)}{5}$$

$$\int x\cos(5x)\ \mathrm{d}x = \frac{x\sin(5x)}{5} - \frac{1}{5}\int \sin(5x)\ \mathrm{d}x$$

$$= \frac{x\sin(5x)}{5} - \frac{1}{5}(-\frac{\cos(5x)}{5}) + C$$

$$= \frac{1}{5}x\sin(5x) + \frac{1}{25}\cos(5x) + C$$

Exercise 11

$$\int t^4\ln(t)\ \mathrm{d}t$$

$$Let : u = \ln(t) \qquad \mathrm{d}(v) = t^4\ \mathrm{d}t$$

$$\mathrm{d}u = \frac{1}{t}\ \mathrm{d}t \quad v = \frac{t^5}{5}$$

$$\int t^4\ln(t)\ \mathrm{d}t = \frac{t^5\ln t}{5} - \int \frac{t^5}{5t}\ \mathrm{d}t$$

$$= \frac{1}{5}t^4\ln(t) - \frac{1}{5}\int t^4\ \mathrm{d}t = \frac{1}{5}t^4\ln(t) - \frac{1}{5}\left[\frac{t^5}{5}\right]$$

$$= \frac{1}{5}t^4\left(\ln(t) - \frac{t}{5}\right)$$

Exercise 17

$$\int e^{2\theta} \sin(3\theta)\ d(\theta)$$

$$Let : u = \sin(3\theta) \qquad dv = e^{2\theta}\ d\theta$$

$$du = 3\cos(3\theta)\ d\theta \quad v = \frac{e^{2\theta}}{2}$$

$$\int e^{2\theta} \sin(3\theta)\ d(\theta) = \frac{e^{2\theta}\sin(3\theta)}{2} - \int \frac{3e^{2\theta}\cos(3\theta)}{2}\ d\theta$$

$$\int e^{2\theta} \sin(3\theta)\ d(\theta) = \frac{e^{2\theta}\sin(3\theta)}{2} - \frac{3}{2}\int e^{2\theta}\cos(3\theta)$$

We must use integration by parts again to separate the integral.

$$\int e^{2\theta}\cos(3\theta)$$

$$Let : u = \cos(3\theta) \qquad dv = e^{2\theta}\ d\theta$$

$$du = -3\sin(3\theta)\ d\theta \quad v = \frac{e^{2\theta}}{2}$$

$$\int e^{2\theta}\cos(3\theta) = \frac{e^{2\theta}\cos(3\theta)}{2} - \int -\frac{3}{2}e^{2\theta}\sin(3\theta)\ d\theta$$

$$= \frac{e^{2\theta}\cos(3\theta)}{2} + \frac{3}{2}\int e^{2\theta}\sin(3\theta)\ d\theta$$

We can form the following equation from this:

$$\int e^{2\theta}\sin(3\theta)\ d\theta = \frac{e^{2\theta}\sin(3\theta)}{2} - \frac{3}{2}\left(\frac{e^{2\theta}\cos(3\theta)}{2} + \frac{3}{2}\int e^{2\theta}\sin(3\theta)\ d\theta\right)$$

$$\int e^{2\theta}\sin(3\theta)\ d\theta = \frac{e^{2\theta}\sin(3\theta)}{2} - \frac{3e^{2\theta}\cos(3\theta)}{4} - \frac{9}{4}\int e^{2\theta}\sin(3\theta)\ d\theta\Big)$$

$$\frac{13}{4}\int e^{2\theta}\sin(3\theta)\ d\theta = \frac{e^{2\theta}\sin(3\theta)}{2} - \frac{3e^{2\theta}\cos(3\theta)}{4}$$

$$= \int e^{2\theta}\sin(3\theta)\ d\theta = \frac{2e^{2\theta}\sin(3\theta)}{13} - \frac{3e^{2\theta}\cos(3\theta)}{13}$$

Exercise 19

$$\int z^3 e^z \, dz$$

$$Let: u = z^3 \quad dv = e^z \, dz$$

$$du = 3z^2 \, dz \quad v = e^z$$

$$\int z^3 e^z \, dz = z^3 e^z - \int 3z^2 e^z = z^3 e^z - 3\int z^2 e^z$$

We can use integration by parts again to reduce the new integral.

$$\int z^2 e^z$$

$$Let: u = z^2 \quad dv = e^z \, dz$$

$$du = 2z \, dz \quad v = e^z$$

$$\int z^2 e^z = z^2 e^z - \int 2z e^z \, dz = z^2 e^z - 2\int z e^z \, dz$$

We can use integration by parts one more time to remove the z from the integral.

$$\int z e^z \, dz$$

$$Let: u = z \quad dv = e^z \, dz$$

$$du = \, dz \quad v = e^z$$

$$\int z e^z \, dz = z e^z - \int e^z \, dz$$

$$= z e^z - e^z$$

Now we put all this shit back together.

$$\int z^3 e^z \, dz = z^3 e^z - 3\int z^2 e^z$$

$$z^3 e^z - 3\left(z^2 e^z - 2\int z e^z \, dz\right)$$

$$z^3 e^z - 3z^2 e^z + 6\int z e^z \, dz$$

$$z^3 e^z - 3z^2 e^z + 6(z e^z - e^z)$$

$$= z^3 e^z - 3z^2 e^z + 6z e^z - 6e^z$$

If you have any questions, comments, or concerns, please contact me at
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