Section 5.5

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Exercise 7

$$\int x\sqrt{1-x^2}\ \mathrm{d}x$$

$$Let: u = 1 - x^2$$

$$\mathrm{d}u = -2x\ \mathrm{d}x$$

$$\int x\sqrt{u}\frac{\mathrm{d}u}{-2x}$$

$$-\frac{1}{2}\int \sqrt{u}\ \mathrm{d}u$$

$$-\frac{1}{2}(\frac{2}{3}u^{\frac{3}{2}})$$

$$= -\frac{1}{3}(1-x^2)^{\frac{2}{3}} + C$$

Exercise 15

$$\int \cos^3(\theta)\sin(\theta)\ \mathrm{d}\theta$$

$$Let: u = \cos(\theta)$$

$$\mathrm{d}u = -\sin(\theta)\ \mathrm{d}\theta$$

$$\int u^3\sin(\theta)\frac{\mathrm{d}u}{-\sin(\theta)}$$

$$-\int u^3\ \mathrm{d}u$$

$$-(\frac{u^4}{4})$$

$$= -\frac{\cos^4\theta}{4} + C$$

Exercise 21

$$\int \frac{(\ln x)^3}{x}\ \mathrm{d}x$$

$$Let : u = \ln x$$

$$\mathrm{d}u = \frac{1}{x}\ \mathrm{d}x$$

$$\int \frac{u^3}{x} x\ \mathrm{d}x$$

$$\int u^3\ \mathrm{d}x$$

$$(\frac{u^4}{4})$$

$$= \frac{(\ln x)^4}{4} + C$$

Exercise 25

$$\int \mathrm{e}^x \sqrt{1+\mathrm{e}^x}\ \mathrm{d}x$$

$$Let : u = 1 + \mathrm{e}^x$$

$$\mathrm{d}u = e^x\ \mathrm{d}x$$

$$\int e^x \sqrt{u} \frac{\mathrm{d}u}{e^x}$$

$$\int \sqrt{u}\ \mathrm{d}u$$

$$\frac{2}{3} u^{\frac{3}{2}}$$

$$= \frac{2}{3}(1+\mathrm{e}^x)^{\frac{3}{2}} + C$$

Exercise 45

$$\int \frac{1+x}{1+x^2}\ \mathrm{d}x$$

$$Let : u = 1 + x^2$$

$$\mathrm{d}u = 2x\ \mathrm{d}x$$

$$\int \frac{1+x}{1+x^2} = \int \frac{1}{1+x^2}\ \mathrm{d}x + \int \frac{x}{1+x^2}\ \mathrm{d}x$$

$$\int \frac{1}{1+x^2}\ \mathrm{d}x + \int \frac{x}{u} \frac{\mathrm{d}u}{2x}$$

$$\int \frac{1}{1+x^2}\ \mathrm{d}x + \frac{1}{2}\int \frac{1}{u}\ \mathrm{d}u$$

$$\tan^{-1}(x) + \frac{1}{2}\ln u$$

$$\tan^{-1}(x) + \frac{1}{2}\ln(1+x^2) + C$$

Exercise 67

$$\int_1^2 x\sqrt{x-1}\ \mathrm{d}x$$

$$Let : u = x - 1$$

$$\mathrm{d}u = \ \mathrm{d}x$$

The integral from 1 to 2 becomes 0 to 1 in terms of u.

$$\int_0^1 (u+1)\sqrt{u}\ \mathrm{d}u$$

$$\int_0^1 u^{\frac{3}{2}} + u^{\frac{1}{2}}\ \mathrm{d}u$$

$$\left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$$

If you have any questions, comments, or concerns, please contact me at
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