Curves Defined By Parametric Equations

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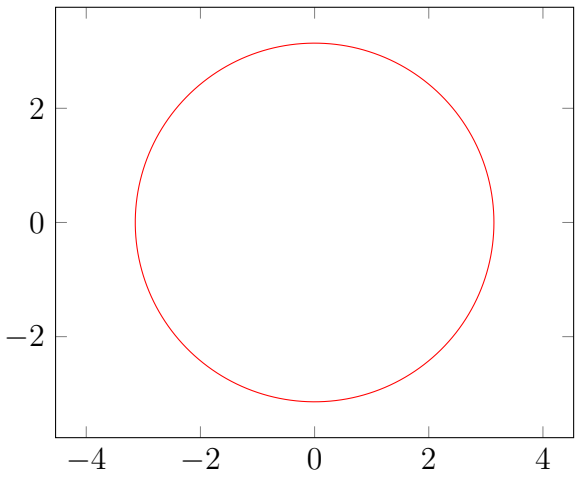
Curves Defined By Parametric Equations

A curve defined by a parametric equation is a curve defined by:

$$y = f(t) \quad y = g(t) \quad t_0 \leq t \leq t_1$$

Such as:

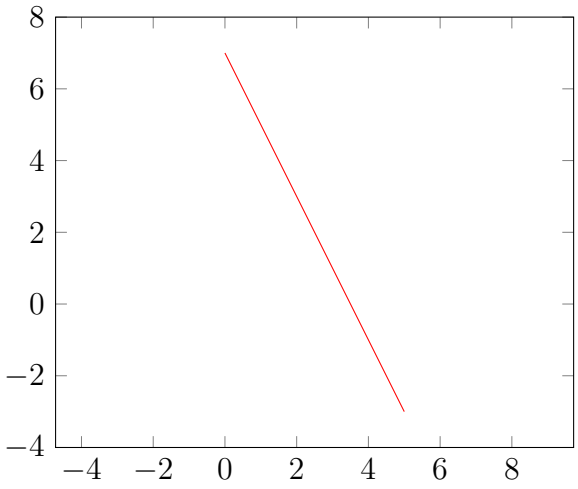


$$x = \pi \cos(\theta) \quad y = \pi \sin(\theta) \quad 0 \leq \theta \leq 2\pi$$

Example 1

Eliminate the parameters and find the equation of the graph in terms of y and x:

$$x = 1 + t \quad y = 5 - 2t \quad -2 \leq t \leq 3$$



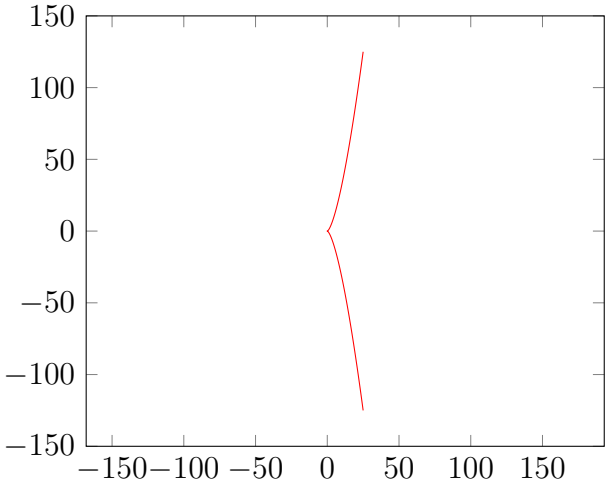
$$t = x - 1$$

$$y = 5 - 2(x - 1)$$
$$y = 5 - 2x + 2$$
$$y = -2x + 7$$

Example 2

Eliminate the parameters and find the equation of the graph in terms of y and x:

$$x = t^2 \quad y = t^3$$

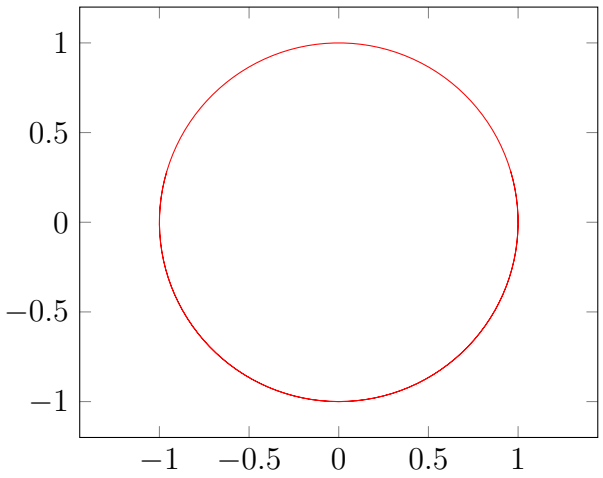


$$y^2 = (t^3)^2 = t^6 = (t^2)^3 = x^3$$
$$y^2 = x^3$$

Example 3

Eliminate the parameters and find the equation of the graph in terms of y and x:

$$x = \sin(\theta) \quad y = \cos(\theta) \quad 0 \leq \theta \leq \pi$$

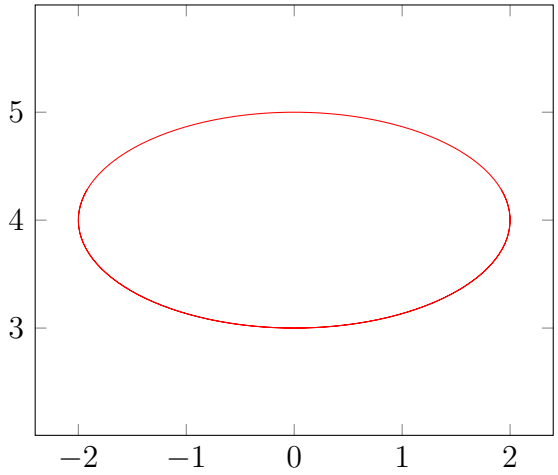


$$x^2 = \sin^2(\theta) \quad y^2 = \cos^2(\theta)$$
$$x^2 + y^2 = \sin^2(\theta) + \cos^2(\theta) = 1$$
$$x^2 + y^2 = 1$$

Example 4

Eliminate the parameters and find the equation of the graph in terms of y and x:

$$x = 2\sin(\theta) \quad y = 4 + \cos(\theta)$$



$$\sin(\theta) = \frac{x}{2} \quad \cos(\theta) = y - 4$$

$$\frac{x^2}{4} + (y-4)^2 = \sin^2(\theta) + \cos^2(\theta) = 1$$

$$\frac{x^2}{4} + (y-4)^2 = 1$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech