Average Value of a Function

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Average Value of a Function

Recall the notion of averaging. Given:

$$x_1, x_2, x_3$$

$$x_{average} = \frac{x_1 + x_2 + x_3}{3}$$

The average value of a function from $x = a$ to $x = b$ is:

$$f_{av} = \frac{1}{b-a} \int_a^b f(x)\ \mathrm{d}x$$

This is a continuous version of the standard average formula, which uses discrete points instead.

The Mean Value Theorem of Integrals

If f is continuous on [a, b], then there exists c in [a, b] such that

$$f(c) = f_{av} = \frac{1}{b-a} \int_a^b f(x)\ \mathrm{d}x$$

Therefore:

$$\int_a^b f(x)\ \mathrm{d}x = (b-a)f(c)$$

Practice Problem 9

$$f(x) = (x-3)^2 \quad [2,5]$$

$$f_{av} = \frac{1}{b-a}\int_a^b f(x)\ \mathrm{d}x$$

$$\frac{1}{5-2}\int_2^5 (x-3)^2\ \mathrm{d}x$$

$$\frac{1}{3}\left[\frac{(x-3)^3}{3}\right]_2^5$$

$$\frac{1}{9}\left[(x-3)^3\right]_2^5$$

$$= \frac{1}{9}(8+1) = 1$$

$$f(c) = f_{av} = 1$$

$$(c-3)^3 = 1$$

$$c^2 - 6c + 9 = 1$$

$$c^2 - 6c + 8 = 1$$

$$(c-2)(c-4) = 1$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech