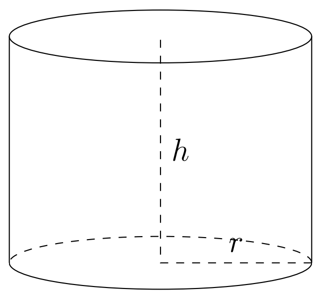
Volumes By Integration (Shells)

Alvin Lin

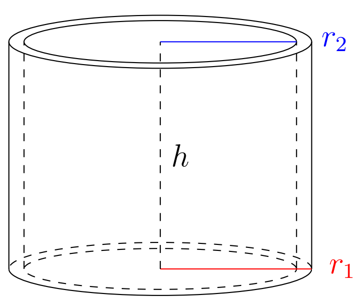
Calculus II: August 2016 - December 2016

Volumes By Integration (Shells)



With the disk method of volume by integration, you can imagine this problem as an integration problem where a cross section of the cylinder is rotated an axis.

There is another method that uses infinitely small shells which compose the volume.



The volume of the disk is:

$$V = \pi((r_1)^2 - (r_2)^2)h$$

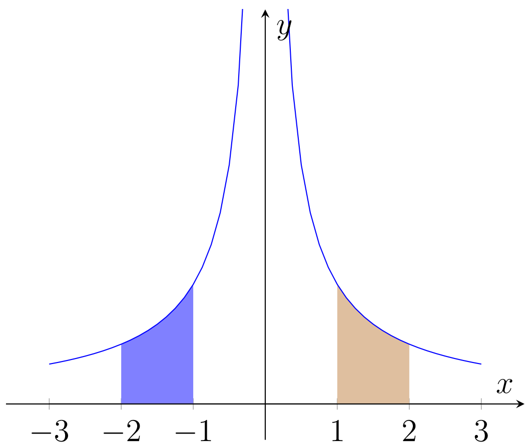
$$V = \lim_{r_1 \to r_2} 2\pi \frac{r_1 + r_2}{2}(r_1 - r_2)h$$
$$V = 2\pi \bar{r}(\Delta r)h$$

And as a general form, the sums of the volumes of all the disks that compose the figure is:

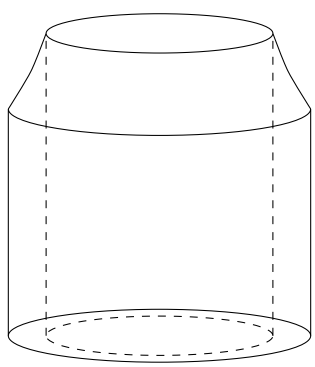
$$V = \int_a^b 2\pi x f(x) \, \mathrm{d}x$$

Example 1

$$y = \frac{1}{x} \quad y = 0 \quad x = 1 \quad x = 2$$



When the red section is rotated about the y-axis, it passes through the highlighted blue section and the following shape results:



$$V = \int_1^2 2\pi x f(x)\ \mathrm{d}x$$

$$V = 2\pi \int_1^2 x \frac{1}{x}\ \mathrm{d}x$$

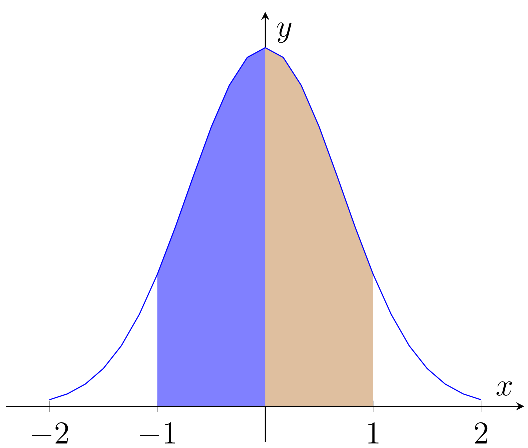
$$V = 2\pi \int_1^2\ \mathrm{d}x$$

$$V = 2\pi \Big[x\Big]_1^2$$

$$V = 2\pi[2-1] = 2\pi$$

Example 2

$$y = \mathrm{e}^{-x^2} \quad y = 0 \quad x = 0 \quad x = 1$$



$$V = \int_0^1 2\pi x f(x)\ \mathrm{d}x$$

$$V = \int_0^1 2\pi x \mathrm{e}^{-x^2}\ \mathrm{d}x$$

$$V = -\pi \int_0^1 -2x\mathrm{e}^{-x^2}\ \mathrm{d}x$$

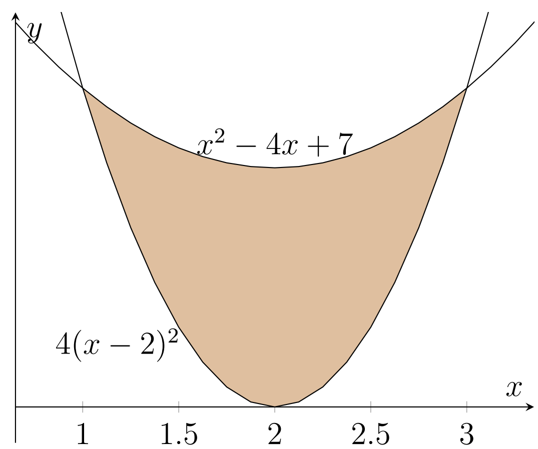
$$V = \pi \Big[\mathrm{e}^{-x^2}\Big]_0^1$$

$$V = \pi\left[e^{0} - e^{-1}\right]$$
$$V = \pi\left[1 - \frac{1}{e}\right]$$
$$V = \pi - \frac{\pi}{e}$$

Practice Problem 7

$$y = 4(x-2)^2 \quad y = x^2 - 4x + 2$$

revolved around the y-axis.



$$V = 2\pi \int_1^3 x\left[(x^2 - 4x + 7) + (4(x-2)^2)\right]\, dx$$
$$V = 2\pi \int_1^3 x\left[x^3 - 4x + 7 - 4x^2 - 16 + 16x\right]\, dx$$
$$V = 2\pi \int_1^3 -3x^3 + 12x^2 - 9x\ dx$$
$$V = 2\pi\left[\frac{-3x^4}{4} + \frac{12x^3}{3} - \frac{9x^2}{2}\right]_1^3$$
$$V = 16\pi$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech