

The Substitution Rule

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Calculus II: August 2016 - December 2016

The Substitution Rule

$$\int \sin(t) \sqrt{1 + \cos(t)} dt$$

Substitute unsolvable terms with u to reduce the equation to something easier to integrate.

$$\text{Let : } u = 1 + \cos(t)$$

$$du = -\sin(t) dt$$

$$dt = \frac{-du}{\sin(t)}$$

Substitute your new terms back into the original equation. The goal is to get a friendlier integral in terms of another variable. If the new integral has both x and u in it, then it may be advisable to try a different method of integration or substituting a different term.

$$\int \sin(t) \sqrt{u} \left(\frac{-du}{\sin(t)} \right)$$

Once we simplify the integral after substituting everything, it becomes much easier to integrate.

$$\begin{aligned} & - \int u^{\frac{1}{2}} du \\ & - \left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ & = -\frac{2}{3} (1 + \cos(t))^{\frac{3}{2}} + C \end{aligned}$$

Practice problem 30

$$\int \frac{\sec^2(x)}{\tan^2(x)} dx$$

$$\text{Let : } u = \tan(x)$$

$$du = \sec^2(x) dx$$

$$\int \frac{du}{u^2}$$

$$\int u^{-2} du$$

$$\frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{\tan(x)} + C$$

Practice problem 31

$$\int \frac{\tan^{-1}(x^2)}{1+x^2} dx$$

$$\text{Let : } u = \tan^{-1}(x)$$

$$du = \frac{1}{1+x^2} dx$$

$$\int u^2 du$$

$$= \frac{u^3}{3} + C$$

Practice problem 42

$$\int \frac{\cos(\ln(t))}{t} dt$$

$$\text{Let : } u = \ln(t)$$

$$du = \frac{1}{t} dt$$

$$\int \cos(u) du$$

$$= \sin(u) + C$$

You can find all my notes at <http://omgimanagerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanagerd.tech