The Substitution Rule

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The Substitution Rule

$$\int \sin(t)\sqrt{1+\cos(t)}\ \mathrm{d}t$$

Substitute unsolvable terms with u to reduce the equation to something easier to integrate.

$$Let: \quad u = 1+\cos(t)$$

$$\mathrm{d}u = -\sin(t)\ \mathrm{d}t$$

$$\mathrm{d}t = \frac{-\ \mathrm{d}u}{\sin(t)}$$

Substitute your new terms back into the original equation. The goal is to get a friendlier integral in terms of another variable. If the new integral has both x and u in it, then it may be advisable to try a different method of integration or substituting a different term.

$$\int \sin(t)\sqrt{u}(\frac{-\ \mathrm{d}u}{\sin(t)})$$

One we simplify the integral after substituting everything, it becomes much easier to integrate.

$$-\int u^{\frac{1}{2}}\ \mathrm{d}u$$

$$-(\frac{u^{\frac{3}{2}}}{\frac{3}{2}})+C$$

$$=-\frac{2}{3}(1+\cos(t))^{\frac{3}{2}}+C$$

Practice problem 30

$$\int \frac{sec^2(x)}{\tan^2(x)}\ \mathrm{d}x$$

$$Let: \quad u = \tan(x)$$

$$\mathrm{d}u = sec^2(x)\ \mathrm{d}x$$

$$\int \frac{\mathrm{d}u}{u^2}$$

$$\int u^{-2}\ \mathrm{d}u$$

$$\frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{\tan(x)} + C$$

Practice problem 31

$$\int \frac{\tan^{-1}(x^2)}{1+x^2}\ \mathrm{d}x$$

$$Let: \quad u = \tan^{-1}(x)$$

$$\mathrm{d}u = \frac{1}{1+x^2}\ \mathrm{d}x$$

$$\int u^2\ \mathrm{d}u$$

$$= \frac{u^2}{3} + C$$

Practice problem 42

$$\int \frac{\cos(ln(t))}{t}\ \mathrm{d}t$$

$$Let: \quad u = ln(t)$$

$$\mathrm{d}u = \frac{1}{t}\ \mathrm{d}t$$

$$\int \cos(u)\ \mathrm{d}u$$

$$= \sin(u) + C$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech