Introduction to Computer Vision

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Photometric Stereo

It is possible to reconstruct a surface from a series of pictures of that surface taken under different illuminations. We only need to obtain measures of the depth of the surface in order to reconstruct its shape. A Monge patch is a representation of a piece of surface as a height function. An orthographic camera that maps (x, y, z) to (x, y) in the camera is viewing a Monge patch. Photometric stereo is a method for recovering a representation of the Monge patch from image data.

Lambert's Law

Recall that

$$B = \rho(\vec{N} \cdot \vec{S}) = \rho\|\vec{S}\| \cos\theta$$

where B is the radiosity (total power leaving the surface per unit area), ρ is the albedo (fraction of incident irradiance reflected by the surface), $\vec{N}$ is the unit normal, and $\vec{S}$ is the source vector whose magnitude is proportional to the intensity of the source.

If we fix the position of the camera and surface and illuminate it using some faraway source, then we can calculate the radiosity as:

$$B(x) = \rho(x)N(x)S(x)$$

The intensity value of a pixel at (x, y) is now:

$$I(x, y) = kB(x)$$

We know the source vectors S_j and the pixel values $I_j(x, y)$ where j is the index of the illumination source. What we need to find is the surface normal $N(x, y)$ and

the albedo $\rho(x, y)$. If we assume that the repsonse function of the camera is a linear scaling by a factor of k, then by Lambert's law:

$$
\begin{aligned}
I_j(x, y) &= kB(x) \\
&= kB(x, y) \\
&= k\rho(x, y)(N(x, y) \cdot S_j) \\
&= (\rho(x, y)N(x, y)) \cdot (kS_j) \\
&= g(x, y) \cdot V_j
\end{aligned}
$$

where $g(x, y)$ describes the surface, V_j is a property of the illumination and camera. By taking the dot product between the vector field $g(x, y)$ and V_j for n sources, we can stack up the known V_j vectors into a matrix V. We can set up the following linear system for each pixel.

$$
\begin{bmatrix} I_1(x, y) \\ I_2(x, y) \\ \vdots \\ I_n(x, y) \end{bmatrix} = \begin{bmatrix} V_1^T \\ V_2^T \\ :V_n^T \end{bmatrix} g(x, y)
$$

We want to obtain the least-squares solution for $g(x, y)$, which we have defined as $N(x, y)\rho(x, y)$. Since $N(x, y)$ is the unit normal, $\rho(x, y)$ is given by the magnitude of $g(x, y)$. Thus, we can also calculate $N(x, y) = \frac{g(x,y)}{\rho(x,y)}$. These methods assume an orthographic camera model, simplistic reflectance and lighting, and cannot support shadows or inter-reflections.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech