

# Introduction to Intelligent Systems: Homework 5

Alvin Lin - Section 1

August 2017 - December 2017

## Problem 1

Given the Wumpus world example from class, suppose the agent has progressed to the point shown in Figure 7.4(a) on page 239, having perceived nothing at [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], [3,1]. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds. (Hint: there are 32 of them). Mark the worlds in which KB is true and those in which each of the following sentences is true:

$\alpha2$  = "There is not a pit in [2,2]"

$\alpha3$  = "There is a wumpus in [1,3]"

Hence show that  $KB \models \alpha2$  and  $KB \models \alpha3$ .

$\alpha2$				$\alpha2, \alpha3$				$\alpha2$				$\alpha2$			
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	W	-	-	-	-	-	-	-	-	-	-	-
AS	-	-	-	AS	-	-	-	AS	W	-	-	AS	-	-	-
-	B	-	-	-	B	-	-	-	B	-	-	-	B	W	-

$\alpha2$				$\alpha2, \alpha3$				$\alpha2$				$\alpha2$			
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P	-	-	-	WP	-	-	-	P	-	-	-	P	-	-	-
AS	-	-	-	AS	-	-	-	AS	W	-	-	AS	-	-	-
-	B	-	-	-	B	-	-	-	B	-	-	-	B	W	-

				$\alpha3$											
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	W	-	-	-	-	-	-	-	-	-	-	-
AS	P	-	-	AS	P	-	-	AS	WP	-	-	AS	P	-	-
-	B	-	-	-	B	-	-	-	B	-	-	-	B	W	-

$\alpha2$				$\alpha2, \alpha3, KB$				$\alpha2$				$\alpha2$			
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	W	-	-	-	-	-	-	-	-	-	-	-
AS	-	-	-	AS	-	-	-	AS	W	-	-	AS	-	-	-
-	B	P	-	-	B	P	-	-	B	P	-	-	B	WP	-

				$\alpha 3$											
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P	-	-	-	WP	-	-	-	P	-	-	-	P	-	-	-
AS	P	-	-	AS	P	-	-	AS	WP	-	-	AS	P	-	-
-	B	-	-	-	B	-	-	-	B	-	-	-	B	W	-

  

$\alpha 2$				$\alpha 2, \alpha 3$				$\alpha 2$				$\alpha 2$			
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P	-	-	-	WP	-	-	-	P	-	-	-	P	-	-	-
AS	-	-	-	AS	-	-	-	AS	W	-	-	AS	-	-	-
-	B	P	-	-	B	P	-	-	B	P	-	-	B	WP	-

  

				$\alpha 3$											
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	W	-	-	-	-	-	-	-	-	-	-	-
AS	P	-	-	AS	P	-	-	AS	WP	-	-	AS	P	-	-
-	B	P	-	-	B	P	-	-	B	P	-	-	B	WP	-

  

				$\alpha 3$											
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P	-	-	-	WP	-	-	-	P	-	-	-	P	-	-	-
AS	P	-	-	AS	P	-	-	AS	WP	-	-	AS	P	-	-
-	B	P	-	-	B	P	-	-	B	P	-	-	B	WP	-

The set of worlds where KB is true is a subset of the set of worlds where  $\alpha 2$  is true and the set of worlds where  $\alpha 3$  is true. Therefore,  $KB \models \alpha 2$  and  $KB \models \alpha 3$ .

## Problem 2

Use a truth table to show that

$$\{p \rightarrow q, (m \rightarrow p \vee q), m\} \models q$$

$p$	$q$	$m$	$p \rightarrow q$	$m \rightarrow p \vee q$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	F
F	F	F	T	T

$q$  is true in all worlds where  $p \rightarrow q$ ,  $m \rightarrow p \vee q$ , and  $m$  are true.

### Problem 3

Use a direct proof (not a proof by contradiction) to show the following.

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \vdash p \rightarrow r \end{array}$$

For each step of the proof, indicate the premise and the logic rule used. Use only the rules from the notes.

$$p \rightarrow q \quad q \rightarrow r \quad \text{Assumptions} \tag{1}$$

$$p \rightarrow q \wedge q \rightarrow r \quad \text{And-Introduction on step 1} \tag{2}$$

$$p \rightarrow r \quad \text{Hypothetical Syllogism on step 2} \tag{3}$$

### Problem 4

Which of the following are correct? If they are incorrect, show the truth assignments that show it. (Hint: Look at page 249 in R&N).

1.  $False \models True$

Correct.  $False \models True$  is correct if and only if  $False \rightarrow True$  is valid.  $False \rightarrow True$  is a tautology so it is valid.

2.  $True \models False$

Incorrect.  $True \models False$  is correct if and only if  $True \rightarrow False$  is valid.  $True \rightarrow False$  is not a tautology since it is always false.

3.  $(A \wedge B) \models (A \Leftrightarrow B)$

$A$	$B$	$A \wedge B$	$A \Leftrightarrow B$	$A \wedge B \rightarrow A \Leftrightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	T	T

Correct, the statement  $A \wedge B \rightarrow A \Leftrightarrow B$  is a tautology and is valid in all models.

4.  $(A \Leftrightarrow B) \models A \vee B$

$A$	$B$	$A \Leftrightarrow B$	$A \wedge B$	$A \wedge B \rightarrow A \Leftrightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	F

Incorrect, the statement  $(A \Leftrightarrow B) \models A \vee B$  is not a tautology. The entailment is incorrect because the implication is false when  $A$  and  $B$  are both false.

5.  $(A \wedge B) \rightarrow C \models (A \rightarrow C) \vee (B \rightarrow C)$

$A$	$B$	$C$	$(A \wedge B) \rightarrow C$	$(A \rightarrow C) \vee (B \rightarrow C)$	$((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Correct, the statement  $((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$  is a tautology.

### Problem 5

Given the following, prove the deduction by (a) a direct proof and (b) a Reductio Ad Absurdum (proof by contradiction). For each step of the proof, indicate the premise and the logic rule used.

$$\begin{aligned}
 &H \rightarrow I \wedge J \rightarrow K \\
 &(I \vee K) \rightarrow L \\
 &\quad \neg L \\
 &\vdash \neg(H \vee J)
 \end{aligned}$$

Direct Proof:

- $(I \vee K) \rightarrow L$  Assumption (1)
- $\neg L \rightarrow \neg(I \vee K)$  Contraposition on step 1 (2)
- $\neg L$  Assumption (3)
- $\neg(I \vee K)$  Modus Ponens using steps 2 and 3 (4)
- $\neg I \wedge \neg K$  De Morgan's Laws on step 7 (5)
- $\neg I$  And-Elimination on step 8 (6)
- $\neg K$  And-Elimination on step 8 (7)
- $H \rightarrow I \wedge J \rightarrow K$  Assumption (8)
- $H \rightarrow I$  And-Elimination on step 11 (9)
- $\neg I \rightarrow \neg H$  Contraposition on step 12 (10)
- $J \rightarrow K$  And-Elimination on step 11 (11)
- $\neg K \rightarrow \neg J$  Contraposition on step 14 (12)
- $\neg H$  Modus Ponens using step 9 and step 13 (13)
- $\neg J$  Modus Ponens using step 10 and step 15 (14)
- $\neg H \wedge \neg J$  And-Introduction using step 16 and step 17 (15)
- $\neg(H \vee J)$  De Morgan's Laws on step 18 (16)

Reductio Ad Absurdum:

$H \vee J$	Assumption	(1)
$(I \vee K) \rightarrow L$	Assumption	(2)
$\neg L \rightarrow \neg(I \vee K)$	Contraposition on step 2	(3)
$\neg L$	Assumption	(4)
$\neg(I \vee K)$	Modus Ponens using step 3 and step 4	(5)
$\neg I \wedge \neg K$	De Morgan's Laws on step 5	(6)
$\neg I$	And-Elimination on step 6	(7)
$\neg K$	And-Elimination on step 6	(8)
$K \rightarrow \perp$	Rewriting step 8	(9)
$H \rightarrow I \wedge J \rightarrow K$	Assumption	(10)
$H \rightarrow I$	And-Elimination on step 10	(11)
$\neg I \rightarrow \neg H$	Contraposition on step 11	(12)
$\neg H$	Modus Ponens using step 7 and step 12	(13)
$J \rightarrow K$	And-Elimination on step 10	(14)
$J$	Disjunctive Syllogism using step 1 and step 13	(15)
$K$	Modus Ponens using step 14 and step 15	(16)
$\perp$	Modus Ponens using step 9 and step 16	(17)
$\neg(H \vee J)$	Reductio Ad Absurdum	(18)

## Problem 6

Convert the following to CNF notation:

*Hint:* implication has a higher precedence than AND or OR.

1.  $C \wedge F \rightarrow \neg B$

$$\begin{aligned} C \wedge F \rightarrow \neg B \\ \neg(C \wedge F) \vee \neg B \\ \neg C \vee \neg F \vee \neg B \end{aligned}$$

2.  $\neg B \rightarrow (C \wedge D \wedge E)$

$$\begin{aligned} \neg B \rightarrow (C \wedge D \wedge E) \\ \neg(\neg B) \vee (C \wedge D \wedge E) \\ B \vee (C \wedge D \wedge E) \\ (B \vee C) \wedge (B \vee D) \wedge (B \vee E) \end{aligned}$$

3.  $(A \vee B) \Leftrightarrow (C \wedge D)$

$$\begin{aligned} (A \vee B) \Leftrightarrow (C \wedge D) \\ ((A \vee B) \rightarrow (C \wedge D)) \wedge ((C \wedge D) \rightarrow (A \vee B)) \\ (\neg(A \vee B) \vee (C \wedge D)) \wedge (\neg(C \wedge D) \vee (A \vee B)) \\ ((\neg A \wedge \neg B) \vee (C \wedge D)) \wedge ((\neg C \vee \neg D) \vee (A \vee B)) \\ (\neg A \vee C) \wedge (\neg B \vee C) \wedge (\neg A \vee D) \wedge (\neg B \vee D) \wedge (A \vee B \vee \neg C \vee \neg D) \end{aligned}$$

If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)