Introduction to Intelligent Systems: Homework 5

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Problem 1

Given the Wumpus world example from class, suppose the agent has progressed to the point shown in Figure 7.4(a) on page 239, having perceived nothing at [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], [3,1]. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds. (Hint: there are 32 of them). Mark the worlds in which KB is true and those in which each of the following sentences is true:
$\alpha 2$ = "There is not a pit in [2,2]"
$\alpha 3$ = "There is a wumpus in [1,3]"
Hence show that $KB \models \alpha 2$ and $KB \models \alpha 3$.

$\alpha 2$			
-	-	-	-
-	-	-	-
AS	-	-	-
-	B	-	-

$\alpha 2, \alpha 3$			
-	-	-	-
W	-	-	-
AS	-	-	-
-	B	-	-

$\alpha 2$			
-	-	-	-
-	-	-	-
AS	W	-	-
-	B	-	-

$\alpha 2$			
-	-	-	-
-	-	-	-
AS	-	-	-
-	B	W	-

$\alpha 2$			
-	-	-	-
P	-	-	-
AS	-	-	-
-	B	-	-

$\alpha 2, \alpha 3$			
-	-	-	-
WP	-	-	-
AS	-	-	-
-	B	-	-

$\alpha 2$			
-	-	-	-
P	-	-	-
AS	W	-	-
-	B	-	-

$\alpha 2$			
-	-	-	-
P	-	-	-
AS	-	-	-
-	B	W	-

-	-	-	-
-	-	-	-
AS	P	-	-
-	B	-	-

$\alpha 3$			
-	-	-	-
W	-	-	-
AS	P	-	-
-	B	-	-

-	-	-	-
-	-	-	-
AS	WP	-	-
-	B	-	-

-	-	-	-
-	-	-	-
AS	P	-	-
-	B	W	-

$\alpha 2$			
-	-	-	-
-	-	-	-
AS	-	-	-
-	B	P	-

$\alpha 2, \alpha 3$,KB			
-	-	-	-
W	-	-	-
AS	-	-	-
-	B	P	-

$\alpha 2$			
-	-	-	-
-	-	-	-
AS	W	-	-
-	B	P	-

$\alpha 2$			
-	-	-	-
-	-	-	-
AS	-	-	-
-	B	WP	-

-	-	-	-
P	-	-	-
AS	P	-	-
-	B	-	-

$\alpha 3$			
-	-	-	-
WP	-	-	-
AS	P	-	-
-	B	-	-

-	-	-	-
P	-	-	-
AS	WP	-	-
-	B	-	-

-	-	-	-
P	-	-	-
AS	P	-	-
-	B	W	-

$\alpha 2$			
-	-	-	-
P	-	-	-
AS	-	-	-
-	B	P	-

$\alpha 2, \alpha 3$			
-	-	-	-
WP	-	-	-
AS	-	-	-
-	B	P	-

$\alpha 2$			
-	-	-	-
P	-	-	-
AS	W	-	-
-	B	P	-

$\alpha 2$			
-	-	-	-
P	-	-	-
AS	-	-	-
-	B	WP	-

-	-	-	-
-	-	-	-
AS	P	-	-
-	B	P	-

$\alpha 3$			
-	-	-	-
W	-	-	-
AS	P	-	-
-	B	P	-

-	-	-	-
-	-	-	-
AS	WP	-	-
-	B	P	-

-	-	-	-
-	-	-	-
AS	P	-	-
-	B	WP	-

-	-	-	-
P	-	-	-
AS	P	-	-
-	B	P	-

$\alpha 3$			
-	-	-	-
WP	-	-	-
AS	P	-	-
-	B	P	-

-	-	-	-
P	-	-	-
AS	WP	-	-
-	B	P	-

-	-	-	-
P	-	-	-
AS	P	-	-
-	B	WP	-

The set of worlds where KB is true is a subset of the set of worlds where $\alpha 2$ is true and the set of worlds where $\alpha 3$ is true. Therefore, $KB \models \alpha 2$ and $KB \models \alpha 3$.

Problem 2

Use a truth table to show that

$$\{p \to q, (m \to p \vee q), m\} \models q$$

p	q	m	$p \to q$	$m \to p \vee q$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	F
F	F	F	T	T

q is true in all worlds where $p \to q$, $m \to p \vee q$, and m are true.

Problem 3

Use a direct proof (not a proof by contradiction) to show the following.

$$p \to q$$
$$q \to r$$
$$\vdash p \to r$$

For each step of the proof, indicate the premise and the logic rule used. Use only the rules from the notes.

$$p \to q \quad q \to r \quad \text{Assumptions} \tag{1}$$
$$p \to q \wedge q \to r \quad \text{And-Introduction on step 1} \tag{2}$$
$$p \to r \quad \text{Hypothetical Syllogism on step 2} \tag{3}$$

Problem 4

Which of the following are correct? If they are incorrect, show the truth assignments that show it. (Hint: Look at page 249 in R&N).

1. $False \models True$
   Correct. $False \models True$ is correct if and only if $False \to True$ is valid. $False \to True$ is a tautology so it is valid.

2. $True \models False$
   Incorrect. $False \models True$ is correct if and only if $True \to False$ is valid. $True \to False$ is not a tautology since it is always false.

3. $(A \wedge B) \models (A \Leftrightarrow B)$

A	B	$A \wedge B$	$A \Leftrightarrow B$	$A \wedge B \to A \Leftrightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	T	T

   Correct, the statement $A \wedge B \to A \Leftrightarrow B$ is a tautology and is valid in all models.

4. $(A \Leftrightarrow B) \models A \vee B$

A	B	$A \Leftrightarrow B$	$A \wedge B$	$A \wedge B \to A \Leftrightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	F

   Incorrect, the statement $(A \Leftrightarrow B) \models A \vee B$ is not a tautology. The entailment is incorrect because the implication is false when A and B are both false.

5. $(A \wedge B) \to C \models (A \to C) \vee (B \to C)$

A	B	C	$(A \wedge B) \rightarrow C$	$(A \rightarrow C) \vee (B \rightarrow C)$	$((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Correct, the statement $((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$ is a tautology.

Problem 5

Given the following, prove the deduction by (a) a direct proof and (b) a Reductio Ad Absurdum (proof by contradiction). For each step of the proof, indicate the premise and the logic rule used.

$$H \rightarrow I \wedge J \rightarrow K$$
$$(I \vee K) \rightarrow L$$
$$\neg L$$
$$\vdash \neg(H \vee J)$$

Direct Proof:

$(I \vee K) \rightarrow L$	Assumption	(1)
$\neg L \rightarrow \neg(I \vee K)$	Contraposition on step 1	(2)
$\neg L$	Assumption	(3)
$\neg(I \vee K)$	Modus Ponens using steps 2 and 3	(4)
$\neg I \wedge \neg K$	De Morgan's Laws on step 7	(5)
$\neg I$	And-Elimination on step 8	(6)
$\neg K$	And-Elimination on step 8	(7)
$H \rightarrow I \wedge J \rightarrow K$	Assumption	(8)
$H \rightarrow I$	And-Elimination on step 11	(9)
$\neg I \rightarrow \neg H$	Contraposition on step 12	(10)
$J \rightarrow K$	And-Elimination on step 11	(11)
$\neg K \rightarrow \neg J$	Contraposition on step 14	(12)
$\neg H$	Modus Ponens using step 9 and step 13	(13)
$\neg J$	Modus Ponens using step 10 and step 15	(14)
$\neg H \wedge \neg J$	And-Introduction using step 16 and step 17	(15)
$\neg(H \vee J)$	De Morgan's Laws on step 18	(16)

Reductio Ad Absurdum:

$$
\begin{aligned}
H \lor J &\quad \text{Assumption} && (1)\\
(I \lor K) \to L &\quad \text{Assumption} && (2)\\
\neg L \to \neg(I \lor K) &\quad \text{Contraposition on step 2} && (3)\\
\neg L &\quad \text{Assumption} && (4)\\
\neg(I \lor K) &\quad \text{Modus Ponens using step 3 and step 4} && (5)\\
\neg I \land \neg K &\quad \text{De Morgan's Laws on step 5} && (6)\\
\neg I &\quad \text{And-Elimination on step 6} && (7)\\
\neg K &\quad \text{And-Elimination on step 6} && (8)\\
K \to \bot &\quad \text{Rewriting step 8} && (9)\\
H \to I \land J \to K &\quad \text{Assumption} && (10)\\
H \to I &\quad \text{And-Elimination on step 10} && (11)\\
\neg I \to \neg H &\quad \text{Contraposition on step 11} && (12)\\
\neg H &\quad \text{Modus Ponens using step 7 and step 12} && (13)\\
J \to K &\quad \text{And-Elimination on step 10} && (14)\\
J &\quad \text{Disjunctive Syllogism using step 1 and step 13} && (15)\\
K &\quad \text{Modus Ponens using step 14 and step 15} && (16)\\
\bot &\quad \text{Modus Ponens using step 9 and step 16} && (17)\\
\neg(H \lor J) &\quad \text{Reductio Ad Absurdum} && (18)
\end{aligned}
$$

Problem 6

Convert the following to CNF notation:
Hint: implication has a higher precedence than AND or OR.

1. $C \land F \to \neg B$

$$C \land F \to \neg B$$
$$\neg(C \land F) \lor \neg B$$
$$\neg C \lor \neg F \lor \neg B$$

2. $\neg B \to (C \land D \land E)$

$$\neg B \to (C \land D \land E)$$
$$\neg(\neg B) \lor (C \land D \land E)$$
$$B \lor (C \land D \land E)$$
$$(B \lor C) \land (B \lor D) \land (B \lor E)$$

3. $(A \lor B) \Leftrightarrow (C \land D)$

$$(A \lor B) \Leftrightarrow (C \land D)$$
$$((A \lor B) \to (C \land D)) \land ((C \land D) \to (A \lor B))$$
$$(\neg(A \lor B) \lor (C \land D)) \land (\neg(C \land D) \lor (A \lor B))$$
$$((\neg A \land \neg B) \lor (C \land D)) \land ((\neg C \lor \neg D) \lor (A \lor B))$$
$$(\neg A \lor C) \land (\neg B \lor C) \land (\neg A \lor D) \land (\neg B \lor D) \land (A \lor B \lor \neg C \lor \neg D)$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech