Intro to Computer Science Theory: Homework 8

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For each of the following problems, determine if the language is regular or not. If it is regular, provide an FA (state transition diagram), NFA (state transition diagram), or RE for it. If it is not regular, prove it via the pumping lemma. (5 points each). All strings are over $\{a, b\}$.

- The set of all strings of odd length where the middle character is b.

  Let this language be L and assume that it is regular. By the pumping lemma, there exists $p \in \mathbb{N}$ such that for all $s \in L$ such that $|s| \geq p$ there exist x, y, z such that $s = xyz, |xy| \leq p, |y| > 0$, and $\forall i \geq 0 (xy^i z \in L)$.

  Let $s = ab^{2p+1}a$. Let x, y, z be such that $s = xyz, |xy| \leq p, |y| > 0$ and for all $i \geq 0, xy^i z \in L$. Since $|xy| \leq p$, the string y must consist of one or more b's.

  Let $i = 2$. By the pumping lemma, $xy^2z \in L$. But $xy^2z = ab^{2p+1-c}b^{2c}a$ for all $c > 0$. Since this can result in even length strings which are clearly not in the language L, this results in a contradiction and thus the language L is not regular.

- The set of all strings of the form xyz, where $|y| > 0$. (x,y, and z may be different characters for each string).

  $$(a \cup b)^+$$

  This language is regular.

- The set of all strings where the number of a's is divisible by the number of b's. Let this language be L and assume that it is regular. By the pumping lemma, there exists $p \in \mathbb{N}$ such that for all $s \in L$ such that $|s| \geq p$ there exist x, y, z such that $s = xyz, |xy| \leq p, |y| > 0$, and $\forall i \geq 0 (xy^i z \in L)$.

Let $s = a^p b^{kp}$. Let x, y, z be such that $s = xyz, |xy| \leq p, |y| > 0$ and for all $i \geq 0, xy^i z \in L$. Since $|xy| \leq p$, the string xy must consist of all a's, and thus the string y must consist of all a's.

Let $i = 2$. By the pumping lemma, $xy^2z \in L$. But $xy^2z = a^{p-c}a^{2c}b^{2kp}$ where $0 < c \leq p$. In order for $xy^2z \in L$ to be true, $p - c + 2c = 2kp$ must be true for all c such that $0 < c \leq p$. It follows that $xy^2z \notin L$. This is a contradiction, thus L is not a regular language.

- The set of all strings where the number of a's is a power of 2. Let this language be L and assume that it is regular. By the pumping lemma, there exists $p \in \mathbb{N}$ such that for all $s \in L$ such that $|s| \geq p$ there exist x, y, z such that $s = xyz, |xy| \leq p, |y| > 0$, and $\forall i \geq 0 (xy^i z \in L)$.

  Let $s = a^{2^p}$. Let x, y, z be such that $s = xyz, |xy| \leq p, |y| > 0$ and for all $i \geq 0, xy^i z \in L$. It follows that the string y must consist of all a's.

  Let $i = 2$. By the pumping lemma, $xy^2z \in L$. Since $|y| > 0$ and $|xy^2z| > 2^p$, it follows that $|xy^2z| = 2^p + c$ where $0 < c \leq p$. By the definition of L, $xy^2z \notin L$. This is a contradiction, therefore the language L is not a regular language.

- The set of all strings with exactly 2 more a's than b's. Let this language be L and assume that it is regular. By the pumping lemma, there exists $p \in \mathbb{N}$ such that for all $s \in L$ such that $|s| \geq p$ there exist x, y, z such that $s = xyz, |xy| \leq p, |y| > 0$, and $\forall i \geq 0 (xy^i z \in L)$.

  Let $s = a^{p+2}b^p$. Let x, y, z be such that $s = xyz, |xy| \leq p, |y| > 0$ and for all $i \geq 0, xy^i z \in L$. Since $|y| > 0$ and $|xy| \leq p$, it follows that xy must be a string of all a's and thus y is a string of only a's.

  Let $i = 2$. By the pumping lemma, $xy^2z \in L$. But $xy^2z = a^{2+p-c}a^{2c}b^p = a^{2+p+c}b^p$ where $0 < c \leq p$. By the definition of L, $xy^2z \notin L$. This is a contradiction and thus the language L is not a regular language.

If you have any questions, comments, or concerns, please contact me at
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