

# Intro to Computer Science Theory: Homework 7

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## Problem 1

For any alphabet  $\Sigma$ , any nonnegative integer  $n$ , and any  $x_1, \dots, x_n \in \Sigma$ , define  $d(x_1 \dots x_n)$  as  $x_1 x_1 \dots x_n x_n$ . For instance  $d(aabab) = aaaabbaabb$ . For any language  $L$ , define  $D(L) = \{d(x) \mid x \in L\}$ . Show via a formal construction FAs that if  $L$  is a regular language, then so is  $D(L)$ . Prove:

$$\forall \text{FAs } M (\exists \text{FA } N (L(N) = D(L(M))))$$

$D(L)$  is accepted by the finite automaton  $M = (Q, \Sigma, \delta, q_0, F)$  such that:

- $Q = \{x_1, x_2 \mid x \in L\} \cup \{r, s\}$
- $\Sigma = \Sigma$
- $\delta : Q \times \Sigma$  is defined on  $(q, z) \in Q \times \Sigma$ :

$$\delta(q, x) = \begin{cases} x_1 & \text{if } q = s \vee q = x_2 \\ x_2 & \text{if } q = x_1 \\ r & \text{if } q \neq x_1 \end{cases}$$

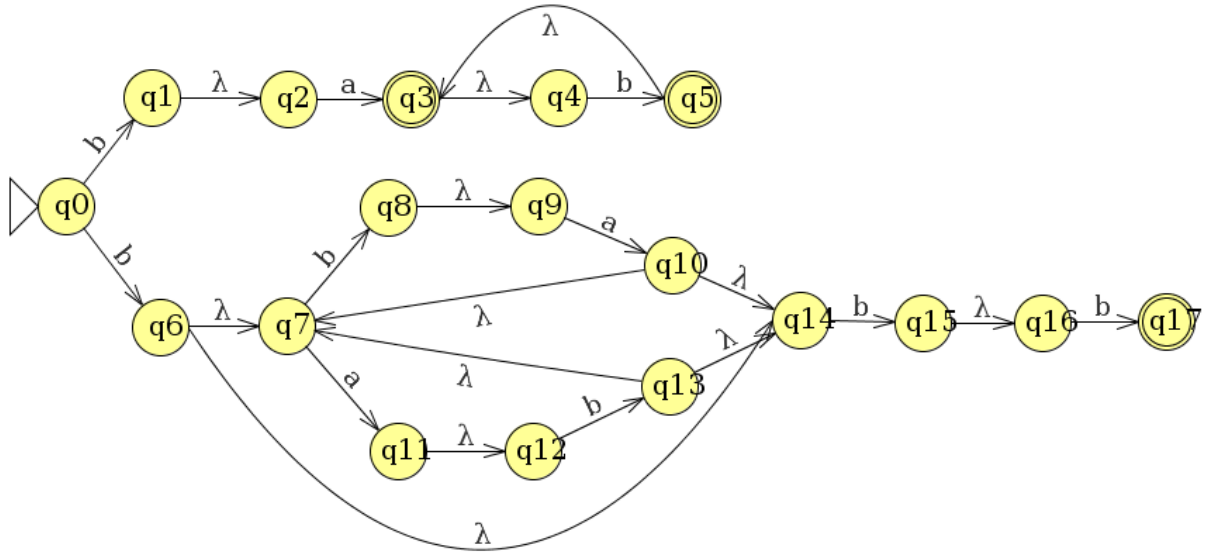
- $q_0 = s$
- $F = \{x_2 \mid x \in L\}$

for any language  $L$  over the alphabet  $\Sigma$ . If  $L$  is a regular language, then  $D(L)$  is also a regular language accepted by the FA above.

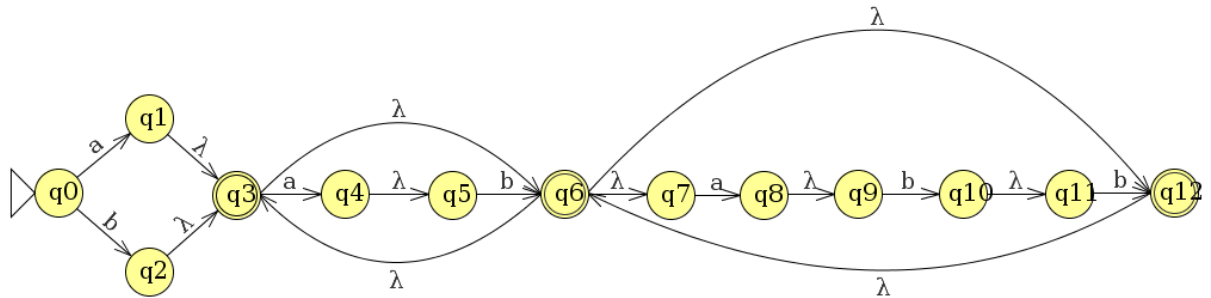
## Problem 2

Use the construction from Lemma 1.55 to make NFAs for the following REs. Your NFAs should be the exact NFA that the construction would produce.

(a)  $bab^* \cup b(ba \cup ab)^*bb$



(b)  $(a \cup b)(ab)^*(abb)^*$



## Problem 3

Use the construction from Lemma 1.60 to make REs for the following DFAs. Your resulting REs should be the exact ones that result from applying the construction

when you eliminate the states in numerical order.

(a)  $b^*aa^* \cup b^*aa^*b(ab^*aa^*b \cup ba^*b)^*ba^*$

(b)  $a(a \cup b) \left( ((a \cup b)^2b) \cup ((a \cup b)^2a(a \cup b)) \right)^* (\epsilon \cup a \cup b)$

If you have any questions, comments, or concerns, please contact me at  
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