Probability and Statistics

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Large Sample Confidence Intervals

For a population mean and proportion:

	population distribution type	sample size	population standard deviation
Section 7.1	normal	any	known
Section 7.2	any	large	unknown

Section 7.1

$$\overline{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

is a large sample confidence interval for μ with confidence level

$$100(1-\alpha)\%$$

$\overline{x}$ represents the observed sample mean, σ represents the population standard deviation, and n represents the sample size.

Section 7.2

Proposition: If n is sufficiently large:

$$Z = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

has approximately a standard normal distribution, with $\overline{X}$ being the random variable for the sample mean, S being the random variable for the sample standard deviation.

A Confidence Interval for a Population Proportion

p : proportion of success in a population

n : sample size

X : # of successes in a sample, a random variable

x : observed # of successes in a sample

$\hat{p} : \dfrac{x}{n}$

$\hat{q} : 1 - \hat{p}$

Assumptions:

- Population size is larger than n
- $np \geq 10$
- $n(1-p) \geq 10$

Proposition:

$$\tilde{p} = \frac{\hat{p} + \frac{(z_{\alpha/2})^2}{2n}}{1 + \frac{(z_{\alpha/2})^2}{n}}$$

The confidence interval for a population proportion p with confidence level $100(1-\alpha)\%$ is:

$$\tilde{p} \pm z_{\alpha/2} \frac{\sqrt{\frac{\hat{p}\hat{q}}{n} + \frac{(z_{\alpha/2})^2}{4n^2}}}{1 + \frac{(z_{\alpha/2})^2}{n}}$$

Under certain conditions, the interval is:

$$\tilde{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech