Probability and Statistics

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Exponential and Gamma Distributions

A random variable X is said to have an exponential distribution with (scale) parameter $\lambda(\lambda > 0)$ if the pdf of X is:

$$f(x;\lambda) = \begin{cases} \lambda \mathrm{e}^{-\lambda x} & , x \geq 0 \\ 0 & , otherwise \end{cases}$$

The cdf of X is:

$$F(x;\lambda) = \begin{cases} 1 - \mathrm{e}^{-\lambda x} & , x \geq 0 \\ 0 & , otherwise \end{cases}$$

The derivative with respect to x of the cdf is equal to the pdf while the parameter λ is fixed.

$$\begin{aligned} F(x;\lambda) &= P(X \leq x) \\ &= \int_{-\infty}^{x} f(t;\lambda)\mathrm{d}t \\ F'(x;\lambda) &= f(x;\lambda) \end{aligned}$$

Expected Values and Variance

Expected value of X:

$$
\begin{aligned}
E(X) &= \int_{-\infty}^{\infty} x f(x;\lambda)\mathrm{d}x \\
&= \int_{0}^{\infty} x\lambda \mathrm{e}^{-\lambda x}\mathrm{d}x \\
&= \frac{1}{\lambda}
\end{aligned}
$$

Variance of X:

$$
\begin{aligned}
V(X) &= \int_{-\infty}^{\infty} (x - E(X))^2 f(x;\lambda)\mathrm{d}x \\
&= \int_{0}^{\infty} (x - \frac{1}{\lambda})^2 \lambda \mathrm{e}^{-\lambda x}\mathrm{d}x \\
&= \int_{0}^{\infty} x^2 \lambda \mathrm{e}^{-\lambda x}\mathrm{d}x - \frac{1}{\lambda^2} \\
&= \frac{1}{\lambda^2} \\
V(X) &= E(X^2) - [E(X)]^2 \\
&= \int_{-\infty}^{\infty} x^2 f(x;\lambda)\mathrm{d}x - (\frac{1}{\lambda})^2 \\
&= \frac{1}{\lambda^2}
\end{aligned}
$$

Relationship between Poisson process and exponential distribution

Suppose that the number of events occurring in any time interval of length t has a Poisson distribution with parameter αt (where α, the rate of the event process, is the expected number of events ocurring in 1 unit of time) and that numbers of occurrences in nonoverlapping intervals are independent of one another. Then the distribution of elapsed time between the occurrence of two successive events is exponential with parameter $\lambda = \alpha$.

If you have any questions, comments, or concerns, please contact me at
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