Probability and Statistics

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Counting Techniques

Product Rule: If the first element or object of an ordered pair can be selected in n_1 ways, and for each of these n_1 ways the second element of the pair can be selected in n_2 ways, then the number of pairs is $n_1 n_2$.

Permutation and Combination Formulas

Number of ways of selecting r items from n distinct items:

$$_nP_r = P(n, r) = P_{r,n} = \frac{n!}{(n-r)!}$$

$$_nC_r = C(n, r) = C_{r,n} = \frac{_nP_r}{r} = \frac{n!}{(n-r)!r!}$$

$$where$$

$$n! = (n)(n-1)(n-2)\dots(3)(2)(1)$$

$$5! = (5)(4)(3)(2)(1)$$

$$3! = (3)(2)(1)$$

$$1! = 1$$

$$0! = 1$$

Repetition is not allowed. For $_nP_r$, order matters, while for $_nC_r$, order does not matter. Suppose we are choosing two colors from R, G, B:

$$_3P_2 = \frac{3!}{(3-2)!} = \frac{(3)(2)(1)}{1} = 6$$

This is analogous to choosing those two colors when order does not matter, thus there are 6 permutations: RG, RB, GR, GB, BR, BG.

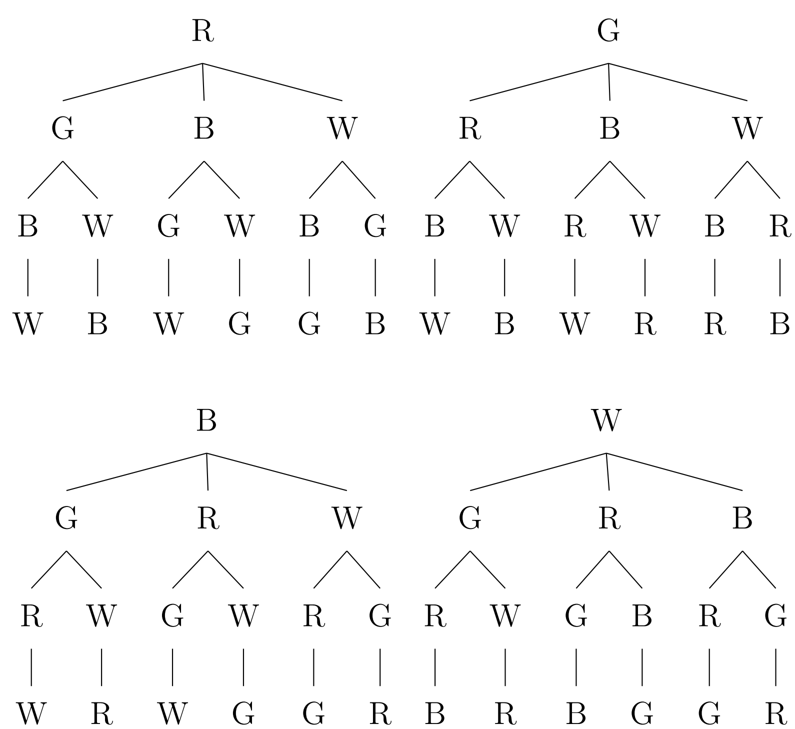
$$_3C_2 = \frac{_3P_2}{2!} = \frac{3!}{(3-2)!2!} = 3$$

This is analogous to choosing those two colors when order does matter, thus there are 3 combinations: RG (same as GR), RB (same as BR), and BG (same as GB).

Example

$$_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 24$$

This is analogous to choosing a permutation of 4 items out of 4 distinct items. Suppose we are choosing 4 colors out of R, G, B, W:



Example

$$_4P_0 = \frac{4!}{(4-0)!} = \frac{4!}{4!} = 1$$

This is analogous to choosing 0 items out of 4 distinct items. We can interpret this as there was no selection and thus no result, resulting in 0 permutations, or we can interpret not selection as an action, thus 1 permutation.

Example

In how many ways can six different cell phones be arranged on top of one another?

$$_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! = 720$$

Example

Consider 5 letters a, b, c, d, e. In how many ways can three letters be selected and arranged if repetition is not allowed?

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

Example

While visiting NYC, the Friedmans are interested in visiting 8 museums but have time to visit only 3. In how many ways can the Friedmans select 3 of the 8 museums to visit?

$$_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = 56$$

Example

A basket contains 4 balls marked R, G, B, Y. Select a ball randomly 2 times with replacement.

$$E_1 : getting\ a\ G\ and\ a\ B\ without\ regard\ to\ order$$
$$E_2 : getting\ two\ Y's$$

Find $P(E_1)$ and $P(E_2)$.

$$P(E_1) \neq \frac{1}{{}_4C_2} = \frac{1}{6}$$

$$P(E_2) \neq \frac{1}{{}_4C_2} = \frac{1}{6}$$

In reality:

$$P(E_1) = \frac{2}{4 \times 4} = \frac{1}{8}$$

$$P(E_2) = \frac{1}{4 \times 4} = \frac{1}{16}$$

Example

Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects. How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?

$${}_{25}C_5 = \frac{25!}{(25-5)!5!} = 53130$$

In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect? The set of A is the set of keyboards with electrical defects, and the set of B is the set of keyboards with mechanical defects.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

No keyboard has both electrical and mechanical defects.
Those with electrical defects: ${}_6C_2$.
Those with mechanical defects: ${}_{19}C_3$.
Answer: ${}_6C_2 \times_{19} C_3 = 984$.
If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect? There are ${}_{25}P_5$ ways to choose 5 keyboards.

$$
\begin{aligned}
P(at\ least\ 4\ mechanical\ defects) &= P(4\ mechanical\ defects \cup 5\ mechanical\ defects) \\
&= P(4\ mechanical\ defects) + P(5\ mechanical\ defects) - \\
&P(4\ mechanical\ defects \cap 5\ mechanical\ defects) \\
&= P(4\ mechanical\ defects) + P(5\ mechanical\ defects) \\
&= \frac{{}_{19}C_4 \times {}_6 C_1}{{}_{25}C_5} + \frac{{}_{19}C_5}{{}_{25}C_5}
\end{aligned}
$$

Every combination has 5! corresponding permutations.

If you have any questions, comments, or concerns, please contact me at
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