

Homework #4

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1

Determine whether each of the following expressions is true or false.

(a) $x \in \{x\}$ True

(b) $\{x\} \in \{\{x\}\}$ True

(c) $\{x\} \subset \{x\}$ True

(d) $\emptyset \subset \{x\}$ True

(e) $\emptyset \in \{x\}$ False

2

Find the power set of each set below. Assume that a and b are distinct elements.

(a) $P(\{a\}) = \{\emptyset, \{a\}\}$

(b) $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

(c) $P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

3

Let $A = \{a, b, c\}$, $B = \{c, d\}$ and $C = \{x, z\}$. Find

(a) $A \times C = \{(a, c), (a, d), (b, c), (b, d), (c, c), (c, d)\}$

- (b) $C \times A \times B = \{(x, a, c), (x, a, d), (x, b, c), (x, b, d), (x, c, c), (x, c, d), (z, a, c), (z, a, d), (z, b, c), (z, b, d), (z, c, c), (z, c, d)\}$
- (c) $A \times B \times C = \{(a, c, x), (a, c, z), (a, d, x), (a, d, z), (b, c, x), (b, c, z), (b, d, x), (b, d, z), (c, c, x), (c, c, z), (c, d, x), (c, d, z)\}$

4

Suppose that

$$A = \{a, b, c, d, e, f\} \text{ and } B = \{a, b, c, d, e, f, g, h, i, j, k\}$$

Find the following

- (a) $A \cup B = \{a, b, c, d, e, f, g, h, i, j, k\}$
- (b) $A - B = \emptyset$
- (c) $A \cap B = \{a, b, c, d, e, f\}$
- (d) $B - A = \{g, h, i, j, k\}$

5

Suppose that A , B , and C are arbitrary sets. Show the following and justify:

- (a) $(A \cup B) \subseteq (A \cup B \cup C)$

$$(A \cup B) = \{x \mid x \in A \vee x \in B\}$$

$$(A \cup B \cup C) = \{x \mid x \in A \vee x \in B \vee x \in C\}$$

Identity Law

- (b) $A - B = A \cap \overline{B}$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \cap \overline{B} = \{x \mid x \in A \wedge x \notin B\}$$

$$(c) (A - B) - C = (A - C) - (B - C)$$

$$(A - B) - C = \{x \mid x \in A \wedge x \notin B \wedge x \notin C\}$$

$$(A - C) = \{x \mid x \in A \wedge x \notin C\}$$

$$(B - C) = \{x \mid x \in B \wedge x \notin C\}$$

$$(A - C) - (B - C) = \{x \mid x \in A \wedge x \notin C\} - \{x \mid x \in B \wedge x \notin C\}$$

$$(A - C) - (B - C) = \{x \mid x \in A \wedge x \notin B \wedge x \notin C\}$$

$$(d) \overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$$

$$\overline{A \cup (B \cap C)} = \overline{A} \cap \overline{B \cap C}$$

$$= \overline{A} \cap (\overline{B} \cup \overline{C})$$

$$= (\overline{B} \cup \overline{C}) \cap \overline{A}$$

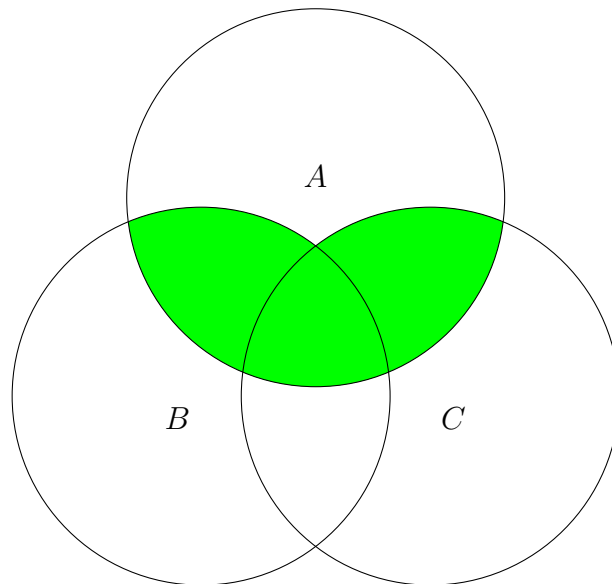
$$= (\overline{C} \cup \overline{B}) \cap \overline{A}$$

6

Suppose that A , B , and C are sets. Draw the Venn diagrams for each of the combinations below.

(a)

$$A \cap (B \cup C)$$



(b)

$$(A - B) \cup (A - C) \cup (B - C)$$

