

Homework #3

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1

Prove or disprove that the product of two irrational integers is irrational. Let \mathbb{Q}' be the set of irrational numbers.

$$\forall p \forall q ((p \in \mathbb{Q}') \wedge (q \in \mathbb{Q}') \wedge (pq = r)) \rightarrow r \in \mathbb{Q}'$$

Contradictory cases:

$$\sqrt{2} \in \mathbb{Q}'; \sqrt{2} \times \sqrt{2} = 2; 2 \in \mathbb{Z}$$

$$\sqrt{2} \in \mathbb{Q}'; \sqrt{3} \in \mathbb{Q}'; \sqrt{2} \times \sqrt{3} = \sqrt{6}; \sqrt{6} \in \mathbb{Q}'$$

The product of two irrational numbers is not always rational.

2

Prove that if n is an integer, then $n^2 \geq n$.

- Case 1: $n = 0$

$$0^2 \geq 0$$

- Case 2: $n > 0$

- Assume $n^2 < n$
- $n < 1$ (Contradiction)
- Therefore, $n^2 \geq n$

- Case 3: $n < 0$

- Assume $n^2 < n$
- $n > 1$ (Contradiction)
- Therefore, $n^2 \geq n$

3

Prove that $\sqrt{5}$ is irrational.

- Assume $\sqrt{5}$ is rational.
- Therefore, $\sqrt{5} = \frac{a}{b}$; $a, b \in \mathbb{Z}; b \neq 0$

$$\begin{aligned}(\sqrt{5})^2 &= \left(\frac{a}{b}\right)^2 \\5 &= \frac{a^2}{b^2} \\5a^2 &= b^2\end{aligned}$$

- Suppose b is even. b^2, a^2, a are all even.
- $b = 2k; a = 2j; k, j \in \mathbb{Z}$
- $\sqrt{5} = \frac{2k}{2j}$ (Contradiction)
- Suppose b is odd. b^2, a^2, a are all odd.
-

$$\begin{aligned}5(2k+1)^2 &= (2j+1)^2; k, j \in \mathbb{Z} \\5(4k^2 + 4k + 1) &= 4j^2 + 4j + 1 \\20k^2 + 20k + 5 &= 4j^2 + 4j + 1 \\20k^2 + 20k + 4 &= 4j^2 + 4j \\5k^2 + 5k + 1 &= j^2 + j \\5k(k+1) + 1 &= j(j+1)\end{aligned}$$

- $5k(k+1) + 1$ is odd and $j(j+1)$ is even. (Contradiction)
- Therefore, $\sqrt{5}$ cannot be rational.

4

What rules of inference are used in the argument: "All men are mortal. Socrates is a man. Thus, Socrates is mortal."?

Modus Ponens

5

What rules of inference are used in the argument: “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”?

Modus Tollens