Homework #2

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1

Alice and Bob are about to play a game of chess, and Alice moves first. If $x_1, \dots, x_n$ represents a sequence of possible moves, we let $W(x_1, \dots, x_n)$ denote the proposition that, after this sequence of moves is completed, Bob is in checkmate.

a

State using quantifier notation the proposition that Alice can force a checkmate on her second move, no matter how Bob plays.

$$\exists x_2(W(x_1, x_2))$$

b

Alice has many possibilities to choose from on her first move, and wants to find one that lets her force a checkmate on her second move. State using quantifier notation the proposition that x_1 is *not* such a move.

$$\neg\exists x_1(W(x_1, x_2))$$

2

State the converse, contrapositive, and inverse for each of the following conditional statements.

a

If it snows tonight, then I will stay at home.
Converse: If I will stay at home, then it will snow tonight.
Contrapositive: If I will not stay at home, then it will not snow tonight.
Inverse: If it will not snow tonight, then I will not stay at home.

b

I go to the beach whenever it is a sunny day.
Converse: It is a sunny day whenever I go to the beach.
Contrapositive: It is not a sunny day whenever I don't go to the beach.
Inverse: I don't go to the beach when it is not a sunny day.

c

A positive integer is a prime only if it has no divisors other than 1 and itself.
Converse: If a positive integer has no divisors other than 1 and itself, it is a prime.
Contrapositive: If a positive integer has divisors other than 1 and itself, then it is not prime.
Inverse: A positive integer is not prime if it has divisors other than 1 and itself.

3

Is the following expression a tautology?

$$\neg p \wedge (p \to q) \to \neg q$$

p	q	$p \to q$	$\neg p \wedge (p \to q)$	$\neg p \wedge (p \to q) \to \neg q$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Nope.

4

Show that $(p \to r) \land (q \to r)$ and $(p \lor q) \to r$ are logically equivalent.

p	q	r	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$p \lor q$	$(p \lor q) \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

5

Find the dual of each of the following compound propositions.

a

$$p \lor \neg q$$
$$s^* \equiv p \land \neg q$$

b

$$p \land (q \lor (r \land T))$$
$$s^* \equiv p \lor (q \land (r \lor F))$$

c

$$(p \vee F) \wedge (q \vee F)$$

$$s^* \equiv (p \wedge T) \vee (q \wedge T)$$

6

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers ($\mathbb{R}$).

a

$$\exists x(x^2 = 2)$$

True when $x = \sqrt{2}$

b

$$\forall x(x^2 + 2 \geq 1)$$

True for all x

c

$$\exists x(x^2 - 2 = 1)$$

True when $x = \sqrt{3}$

d

$$\forall x(x^2 \neq x)$$

False when $x = 1$