

Sets

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Set Operations

Union

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

Let $A = \{1, 2\}$ $B = \{3, 4\}$:

$$A \cup B = \{1, 2, 3, 4\}$$

Intersection

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

Let $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$:

$$A \cap B = \{3\}$$

Set Difference

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

Let $A = \{a, b, c\}$ $B = \{b, c, d\}$:

$$A - B = \{a\}$$

Set Complements

Let u be the universal set. The **complement** of the set A , denoted \bar{A} , is the set $u - A$. An element belongs in \bar{A} if $x \notin A$.

Set Identities

1. Identity Laws

- $A \cap u = A$
- $A \cup \emptyset = A$

2. Domination

- $A \cup u = u$
- $A \cap \emptyset = \emptyset$

3. Idempotent

- $A \cup A = A$
- $A \cap A = A$

4. Complementation

- $\overline{\overline{A}} = A$

5. Commutative

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

6. Associative

- $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cap C) = (A \cap B) \cap C$

7. Distribution

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

8. De Morgan's

- $\overline{A \cap B} = \overline{A} \cup \overline{B}$
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$

9. Absorption

- $A \cup (A \cap B) = A$
- $A \cap (A \cup B) = A$

10. Complement

- $A \cup \bar{A} = u$
- $A \cap \bar{A} = \emptyset$

Example

Show using set identities that:

$$\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$$

$$\begin{aligned}\overline{A \cup (B \cap C)} &= \bar{A} \cap \overline{B \cap C} \quad (\text{De Morgan's}) \\ &= \bar{A} \cap (\bar{B} \cup \bar{C}) \quad (\text{De Morgan's}) \\ &= (\bar{B} \cup \bar{C}) \cap \bar{A} \quad (\text{Commutative}) \\ &= (\bar{C} \cup \bar{B}) \cap \bar{A} \quad (\text{Commutative})\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech