Sets

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Sets

Sets are fundamental discrete structures. A set is an unordered collection of objects called **elements** of the set. A set is said to contain its elements. We write $a \in A$ if a is an element of the set A. We write $a \notin A$ if a is not an element of the set a. Sets are usually given in uppercase letters while elements are generally given in lowercase letters.

There are several ways to describe sets:

1. Roster method
2. Set builder notation
3. Venn diagrams

Roster Notation

The set of all vowels is:

$$V = \left\{a, e, i, o, u\right\}$$

The set of all odd positive integers less than 12 is:

$$O = \left\{1, 3, 5, 7, 9, 11\right\}$$

Other examples:

$$U = \left\{dog, cat, fish\right\}$$

$$dog \in U$$

$$chicken \notin U$$

Set Builder Notation

$$O = \left\{ x | x \textit{ is an odd positive integer less than } 100 \right\}$$

$$[a, b] = \left\{ x | a \leq x \leq b \right\}$$

$$(a, b] = \left\{ x | a < x \leq b \right\}$$

$$[a, b) = \left\{ x | a \leq x < b \right\}$$

$$(a, b) = \left\{ x | a < x < b \right\}$$

Set Equality

Two sets are equal if and only if they contain the same elements. You must show that the two sets are subsets of each other. Thus A and B are equal if:

$$\forall x (x \in A \leftrightarrow x \in B)$$

or

$$A \subseteq B \wedge B \subseteq A$$

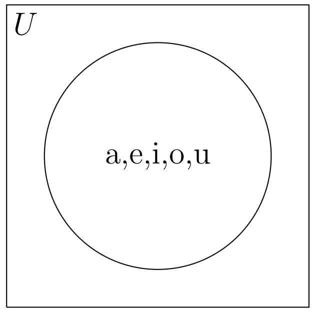
We write $A = B$ when sets are equal.

$$\left\{ 1, 2, 3 \right\} = \left\{ 3, 1, 2 \right\}$$

$$\left\{ 1, 2, 3 \right\} \neq \left\{ 1, 2, 4 \right\}$$

Venn Diagrams

Venn Diagrams are useful for showing relations between sets. The universal set U contains all elements. The set V contains the set of all vowels.



The Empty Set

The empty set $\varnothing$ is the set consisting of no elements (null set).

Theorem: If S is a set, then $\varnothing \subseteq S$ and $S \subseteq S$.

Subsets

The set A is a subset of the set B if and only if every element of A is also in B, notated as $A \subseteq B$

$$\forall x(x \in A \to x \in B)$$

$$\left\{1, 2, 3\right\} \subseteq \left\{1, 2, 3, 4\right\}$$

$$\left\{1, 2, 4\right\} \subseteq \left\{1, 2, 5\right\}$$

Sets may have other sets as members:

$$A = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$$

$$a \notin A, b \notin A$$

$$\{a\} \in A, \{b\} \in A$$

The Size of a Set

Let S be a set. If there are exactly n distinct elements of S, then we say that the <u>size</u> of S is n. Written as $|S| = n$. It is also called the **cardinality** of the set. In this case, S is finite.

Let A be the set of letters in the alphabet. $|A| = 26$.

A set is said to be infinite if it is not finite. For example, the set of integers is infinite.

Power Sets

Given a set S, the power set of S is the set of all subsets of S. This is denoted by P(S).

$$P(\{0,1,2\}) = \{\varnothing, \{0\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

For a set S with $|S| = n$, $|P(S)| = 2^n$.

Cartesian Product

Let A and B be sets. The **Cartesian Product** of A and B, denoted $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Let $A = \{1,2\} \quad B = \{a,b,c\}$:

$$A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)\}$$

$$B \times A = \{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$$

$A \times B \neq B \times A$ since the Cartesian Product is ordered. $(1,a) \in A \times B$ but $(1,a) \notin B \times A$. Also note that the cardinality of a Cartesian Product is as such:

$$|A \times B| = |A||B|$$

If you have any questions, comments, or concerns, please contact me at
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