# Rules of Inference

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## Rules of Inference

Proofs in mathematics are valid arguments that establish the truth of mathematical statements.
**Argument:** Sequence of statements that ends with a conclusion.
A conclusion is **valid** if it follows from the truth of the preceding statements (called premises). Rules of inference are basic tools for establishing the truth of statements.

### Example

Consider the following:

- "If you have a current password, then you can log on to the network"
- "You have a current password"
- "Therefore, you can log onto the network"

Determine whether this is a valid argument. Does the conclusion follow?
Let:

- $p$: you have a current password
- $q$: you can log onto the network

$$p \rightarrow q$$
$$p$$
$$\therefore q$$

$(p \leftrightarrow q \land p) \rightarrow q$ is a tautology, so the argument is valid.

## Example

Suppose we replace $p$ and $q$ by:

- $p$: you have access to the network
- $q$: you can change your grade

$$p : True$$

$$p \rightarrow q : False$$

$p \rightarrow q$ is valid still, but since the premise $p \rightarrow q$ is false, you can't conclude $q$. $(p \wedge (p \rightarrow q)) \rightarrow q$ is a rule of inference called Modus Ponens. It is always valid, but the conclusion might not always be true.

## Rules of Inference

1. Modus Ponens

$$p$$

$$p \rightarrow q$$

$$\therefore q$$

2. Modus Tollens

$$\neg q$$

$$p \rightarrow q$$

$$\therefore \neg p$$

3. Hypothetical Syllogism

$$p \rightarrow q$$

$$q \rightarrow r$$

$$p \rightarrow r$$

4. Disjunctive Syllogism

$$p \vee q$$

$$\neg p$$

$$\therefore q$$

5. Addition

$$p$$
$$\therefore p \vee q$$

6. Simplification

$$p \wedge q$$
$$\therefore p$$

7. Conjunction

$$p$$
$$q$$
$$\therefore p \wedge q$$

8. Resolution

$$p \vee q$$
$$\neg p \vee r$$
$$q \vee r$$

## Example

Determine whether the argument is valid and whether its conclusion must be true. If $\sqrt{2} > \frac{3}{2}$ then $(\sqrt{2})^2 > (\frac{3}{2})^2$. Let:

$$p : \sqrt{2} > \frac{3}{2}$$

$$q : (\sqrt{2})^2 > (\frac{3}{2})^2$$

The conclusion is not true since one premise, namely $\sqrt{2} > \frac{3}{2}$ is false.

## Example

What rules are used?

1. It is sunny now. Therefore, it is either sunny or 32°F.
   **Addition Rule**

2. It is sunny and above 32°F now. Therefore, it is sunny now.
   **Simplification Rule**

3. If it rains today, we will not have a bbq. If we do not have a bbq, then we will tomorrow.
   **Hypothetical Syllogism**

If you have any questions, comments, or concerns, please contact me at
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