Propositional Equivalence

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Discrete Math for Computing: January 2017 - May 2017

Propositional Equivalence

$p \to q$ and $\neg q \to \neg p$ have the same truth table. An implication and its contrapositive are always logically equivalent. A compound proposition is a:

1. **tautology** if it is always true.
2. **contradiction** if it is always false.
3. **contingency** if for some values it is true and some it is false.

$p \vee \neg p$ is a tautology. $p \wedge \neg p$ is a contradiction.

Logical Equivalence

The compound propositions p and q are logically equivalent is $p \leftrightarrow q$ is a tautology.
Notation: We write $p \equiv q$ if p and q are logically equivalent.

$$p \to q \equiv \neg q \vee q$$

$$p \to q \equiv \neg q \to \neg p$$

$$p \vee q \equiv \neg p \to q$$

$$p \wedge q \equiv \neg(p \to \neg q)$$

De Morgan's Laws

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	F	F
T	F	T	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Example

Show that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$:

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Laws of Logical Equivalence

1. Identity Law:
   - $p \wedge T \equiv p$
   - $p \vee F \equiv p$
2. Domination Law:
   - $p \vee T \equiv T$
   - $p \wedge F \equiv F$
3. Idempotent Law:

- $p \lor p \equiv p$
- $p \land p \equiv p$

4. Double Negation:
   - $\neg(\neg p) \equiv p$
5. Commutative Laws:
   - $p \lor q \equiv q \lor p$
   - $p \land q \equiv q \land p$
6. Associative Laws:
   - $(p \lor q) \lor r \equiv p \lor (q \lor r)$
   - $(p \land q) \land r \equiv p \land (q \land r)$
7. Distributive Laws:
   - $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
   - $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
8. De Morgan's Laws:
   - $\neg(p \land q) \equiv \neg p \lor \neg q$
   - $\neg(p \lor q) \equiv \neg p \land \neg q$
9. Absorption Laws:
   - $p \lor (p \land q) \equiv p$
   - $p \land (p \lor q) \equiv p$
10. Negation:
    - $p \lor \neg p \equiv T$
    - $p \land \neg p \equiv F$

Example

Show $\neg(p \to q)$ and $p \land \neg q$ are logically equivalent using the laws.
Hint: $p \to q \equiv \neg p \lor q$

$$\begin{aligned} p \to q &\equiv \neg p \lor q \\ &\equiv \neg(\neg p \lor q) \\ &\equiv \neg(\neg p) \lor \neg q \\ &\equiv p \lor \neg q \end{aligned}$$

If you have any questions, comments, or concerns, please contact me at
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